

Your Name

Your Signature

Student ID #

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Quiz Section

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Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes. Do not share notes.
- Give your answers in exact form. Do not give decimal approximations.
- Graphing calculators are not allowed.
- In order to receive credit, you must *show all of your work*. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

| Problem | Total Points | Score |
|---------|--------------|-------|
| 1 | 14 | |
| 2 | 14 | |
| 3 | 7 | |
| 4 | 9 | |
| 5 | 8 | |
| 6 | 8 | |

| Problem | Total Points | Score |
|---------|--------------|-------|
| 7 | 8 | |
| 8 | 7 | |
| 9 | 7 | |
| 10 | 7 | |
| 11 | 11 | |
| Total | 100 | |

1. [14 points total] Evaluate the following indefinite integrals.

(a) [7 points] $\int \frac{3x}{x^2 - x - 6} dx$

(b) [7 points] $\int x \tan(x) \sec^2(x) dx$

2. [14 points total] Evaluate the following definite integrals.

(a) [7 points] $\int_0^{\pi/2} e^x \sin x dx$

(b) [7 points] $\int_1^2 \frac{1}{x(\ln(3x))^4} dx$

3. [7 points] Does the following improper integral converge? If so, evaluate it. If not, explain why.

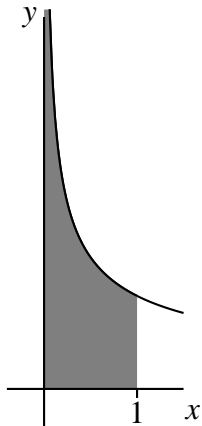
$$\int_0^1 \frac{x+3}{\sqrt{1-x^2}} dx$$

4. [9 points]

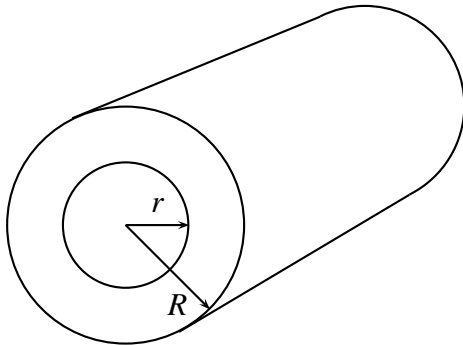
The curve

$$y = \frac{1}{\sqrt{x}} \quad \text{for} \quad 0 < x \leq 1$$

and the positive x - and y -axes determine an *unbounded* region in the plane. Rotate this region about the y -axis, and determine whether the volume of the resulting solid is finite or infinite. If it is finite, find the volume. If it is infinite, explain why.



5. [8 points] A cylindrical tank with radius R lies on its side, and it has a circular hatch at one end. The hatch has radius r and is centered at the center line of the tank. Calculate the total force acting on the hatch when the tank is full of liquid. In your answer, denote the acceleration due to gravity by g and the density of the liquid by ρ .



6. [8 points] Consider the function $f(x) = \int_0^x \cos(\pi t^2) dt$. For which values of x between -1 and 1 is $f(x)$ increasing?

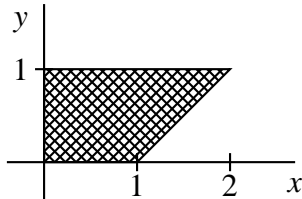
7. [8 points] Solve the initial-value problem $\frac{dy}{dx} = \frac{xy+y}{2\ln y}$, $y(1) = e^2$.

8. [7 points] Find the area of the region enclosed by the parabolas $y = x^2 + 2x$ and $y = -x^2 + 12$.

9. [7 points] A car travels along a straight highway. The following table contains sample values of the velocity of the car from $t = 0$ to $t = 1$ hour, sampled every 10 minutes. Use Simpson's Rule with $n=6$ to approximate the position of the car after 1 hour.

| t (hrs.) | v(t) (mph) |
|----------|------------|
| 0 | 60 |
| 1/6 | 40 |
| 2/6 | -20 |
| 3/6 | -40 |
| 4/6 | -30 |
| 5/6 | 10 |
| 1 | 60 |

10. [7 points] Find the coordinates (\bar{x}, \bar{y}) of the center of mass of the quadrilateral with corners at $(0, 0)$, $(1, 0)$, $(0, 1)$ and $(2, 1)$ pictured here.



11. [11 points total] Lake Pristine has a volume of 1000 cubic meters and contains pure water. A farm near the lake begins using a pesticide. Runoff from the farmland into the lake is 10 cubic meters of water per day, with a concentration of 50 grams of pesticide per cubic meter of water.

The lake drains to the ocean at a rate 10 cubic meters per day. You may assume that the pesticide mixes thoroughly with the water in the lake, and you should ignore other effects such as evaporation.

(a) **[6 points]** Let $y(t)$ denote the total amount of pesticide (in grams) in the lake after t days. Set up a differential equation for $y'(t)$.

(b) **[5 points]** Fish can survive a concentration of 1 gram of pesticide per cubic meter of water. Solve the differential equation you found in part (a) and determine whether the fish will be alive after 10 days.