

Your Name

Your Signature

Student ID #

--	--	--	--	--	--	--

Quiz Section

--	--

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2}'' \times 11''$ sheet of handwritten notes (both sides). Do not share notes.
- Give your answers in exact form, except as noted in particular problems.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may use any of the 20 integrals from the table on p. 506 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 11 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	14	
2	6	
3	6	
4	7	
5	7	
6	8	

Question	Points	Score
7	8	
8	10	
9	10	
10	14	
11	10	
Total	100	

1. (14 total points) Evaluate the following integrals.

(a) (7 points) $\int \frac{5x}{(x-1)(x^2+4)} dx$

(b) (7 points) $\int \frac{\cos x}{\sin^2 x - 4 \sin x + 7} dx$

2. (6 points) Let $f(x) = \int_0^{x^2-6x} \frac{2\sqrt{t+9}}{t+16} dt$. Find $f'(6)$.

3. (6 points) Solve the initial value problem. Find an explicit formula for y as a function of x .

$$y' = x^2 + x^2y^2, \quad y(0) = 1.$$

4. (7 points) Evaluate

$$\int_0^{\pi} x \sin^2 x \cos x \, dx.$$

5. (7 points) Evaluate

$$\int \frac{x^3}{\sqrt{x^2 - 9}} dx.$$

6. (8 points) Determine whether the integral

$$\int_0^{\infty} \frac{x}{(x+8)^{7/3}} dx$$

converges, and evaluate it if it does.

7. (8 total points) Two drivers enter a freeway side by side, on parallel lanes. They stop briefly at a light controlling the traffic flow to the freeway. Then they both accelerate over the next 15 seconds, but in different ways. The acceleration of driver A is constant over the first 15 seconds. His speed at the end of this time interval is 80 ft/sec (about 55 MPH). The speed (not acceleration) of driver B over the first 15 seconds is given by $v_B(t) = 80 \sin(t\pi/30)$ ft/sec.

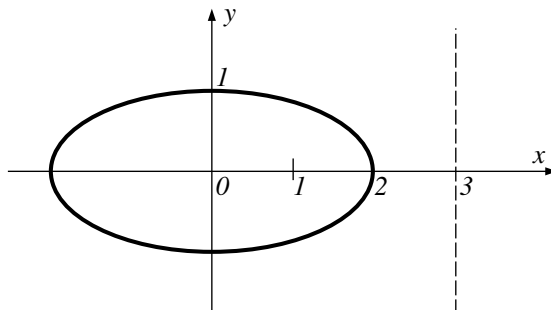
(a) (6 points) Find the distance between drivers A and B at the end of the 15 second interval.

(b) (2 points) Find the average velocity over the 15 second interval for each of the drivers.

8. (10 total points) A bagel is formed by rotating the region inside the ellipse

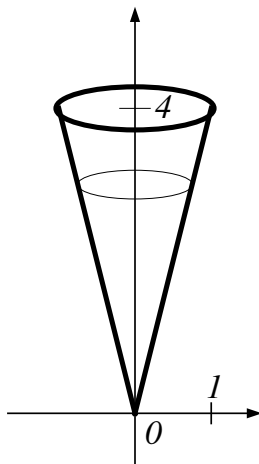
$$\frac{x^2}{4} + y^2 = 1$$

about the vertical line $x = 3$.



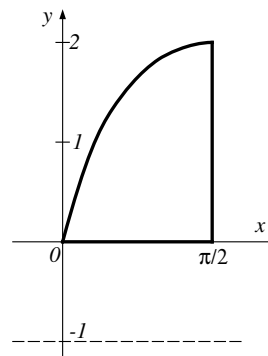
- (a) (1 point) Clearly show, in the drawing above, a typical rectangle in this region that will be rotated to form a cylindrical shell.
- (b) (4 points) Express the volume of the bagel as an integral using the shell method.
- (c) (5 points) Evaluate this integral.

9. (10 points) A conical tank is 4 meters high and has a radius of 1 meter at the top (see picture). The bottom 3 meters are full of water. How much work is required to pump out all the water over the rim of the tank? (Recall that the mass density of water is 1000kg/m^3 and the acceleration due to gravity is 9.8m/sec^2 .)



10. (14 total points) Let \mathcal{R} be the region in the first quadrant enclosed by the graph of the function $y = 2 \sin x$, the x -axis, and the vertical line $x = \pi/2$, as shown in the picture.

(a) (2 points) Find the area of \mathcal{R} .



(b) (8 points) Find the coordinates (\bar{x}, \bar{y}) of the centroid of \mathcal{R} .

(c) (4 points) Find the volume of the solid obtained by rotating the region \mathcal{R} around the horizontal line $y = -1$.

11. (10 total points) The population P of Springfield is increasing steadily, for two reasons: (1) The birth rate exceeds the death rate, resulting in an increase of 2% per year (that is, a rate of increase equal to $.02P$ per year). (2) More people move to Springfield from elsewhere than move away, resulting in an additional increase of 1000 people per year.

(a) (2 points) Write down a formula for the total rate of increase of population, dP/dt .

(b) (6 points) At the beginning of 1995 (call this time $t = 0$), the population of Springfield was 25,000. Solve the differential equation you obtained in part (a) to find the population of Springfield t years later.

(c) (2 points) In what year will the population of Springfield be 60,000? (Your answer should look like “2009”, not like “14.217689...”.)