Math 125 **Final Examination** **Autumn 2008**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- A scientific calculator is allowed, but graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- You may use any of the 20 integrals in the table on p. 484 of the text (p. 506 if you have the 5th edition of Stewart) without deriving them. **Show your work in evaluating any other integrals, even if they are on your note sheet**.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	10	
4	10	
5	10	

Question	Points	Score
6	8	
7	8	
8	10	
9	10	
10	10	
Total	100	

1. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\int \cos^4 x\, dx$

   (b) (6 points) $\int_0^{\pi/4} \frac{\sin t}{\cos^2 t}\, dt$

2. (12 total points) Evaluate the following integrals.

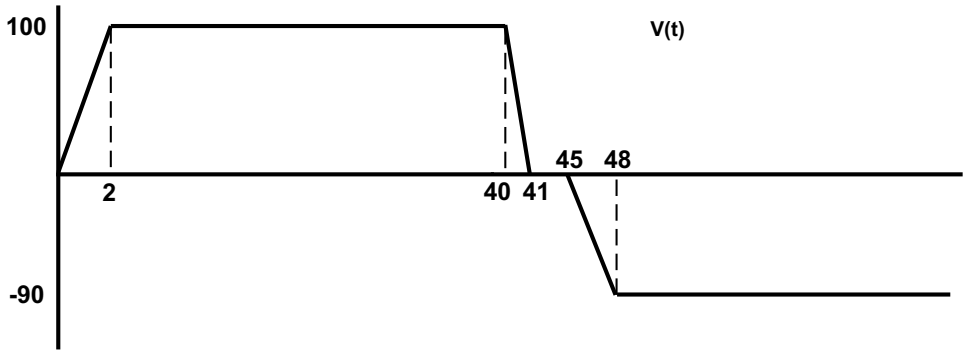
   (a) (6 points) $\int x \ln(x+4)\, dx$

   (b) (6 points) $\int_2^\infty \frac{1}{x\sqrt{x-2}}\, dx$

3. (10 total points) Every morning, Tom runs on a straight line from his house to Fred's house. When he reaches Fred's house, he stops for a break. Then he turns around and runs back to his house. Assume that Tom's velocity this morning, measured in meters per minute, was given by

$$v(t) = \begin{cases} 50t & \text{if } 0 \le t \le 2, \\ 100 & \text{if } 2 \le t \le 40, \\ -100t + 4100 & \text{if } 40 \le t \le 41, \\ 0 & \text{if } 41 \le t \le 45, \\ 1350 - 30t & \text{if } 45 \le t \le 48, \\ -90 & \text{if } t \ge 48 \text{ until he reaches home,} \end{cases}$$

where t is time (in minutes) after 7:00am. Here is a graph of $v(t)$ (not to scale):



(a) (4 points) Find Tom's average velocity during the first 5 minutes of his run.

(b) (3 points) How far is Fred's house from Tom's house?

(c) (3 points) What is the total distance Tom covers from 7:00am to 7:50am?

4. (10 points) Let $\mathscr{R}$ be the region in the first quadrant bounded by $y = x^2$, $y = 25$, and the y-axis. Find the value of m with the property that the line $y = mx$ divides $\mathscr{R}$ into two regions with the same area.

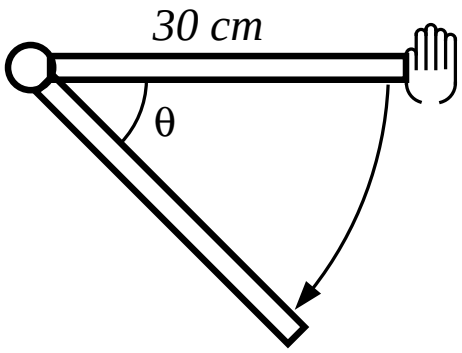
5. (10 points) Compute the volume of the solid obtained by rotating the region bounded between the curves $y = \sin x$ and $y = -\sin x$ on the interval $\pi \leq x \leq 2\pi$ about the y-axis.

6. (8 points) A worker has to tighten a bolt using a wrench. She holds the wrench 30 cm away from the bolt. The more she tightens the bolt, the more force she has to use. The force she applies is equal to $3+\tan^2\theta$ newtons, where θ is the angle between the original position of the wrench and the current position, in radians.

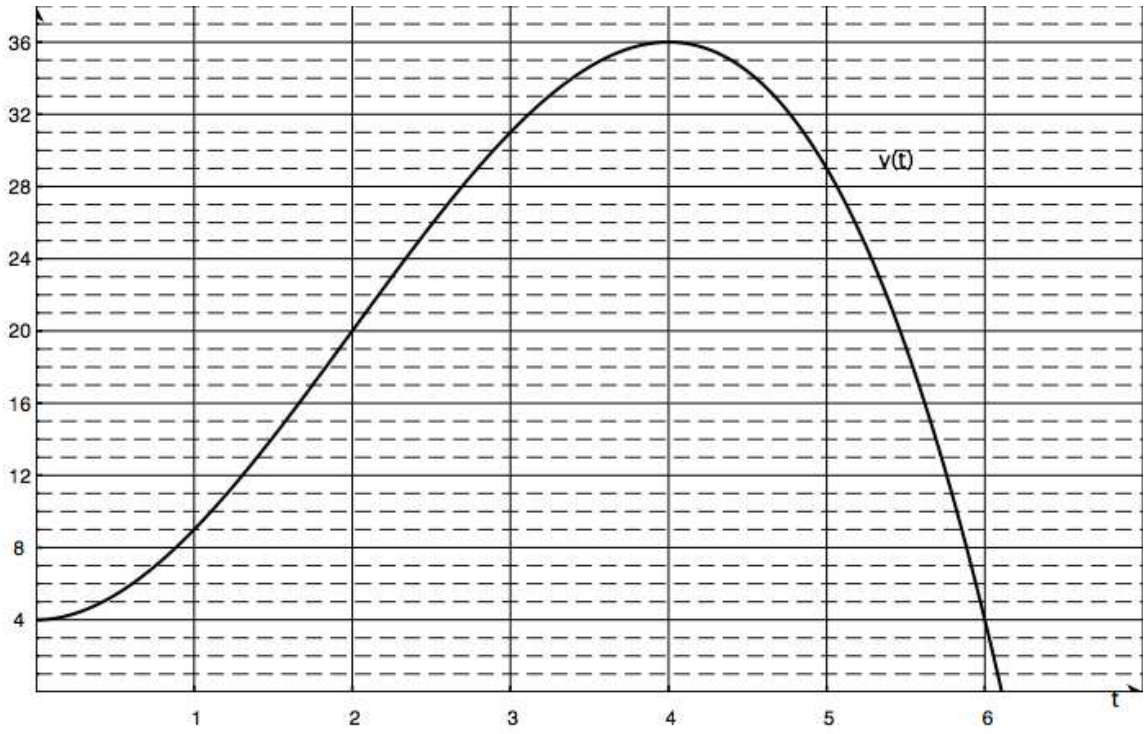
   How much work does she do to turn the bolt $1/8$ of a full turn (that is, $\pi/4$ radians)?

   Give your answer in joules (1 joule = 1 newton-meter).

   (Hint: The distance traveled along a circle of radius r in moving an angle of $\Delta\theta$ radians is $r\Delta\theta$.)



7. (8 points) The graph below shows the velocity $v(t)$ (in meters per second) of an object moving along a straight line as a function of time t (in seconds).



Use Simpson's Rule with $n = 6$ to approximate the distance the object travels from $t = 0$ to $t = 6$.

8. (10 points) Find the centroid of the region which is inside the circle $x^2 + (y+1)^2 = 4$ and above the x-axis.

   Give your answer(s) in exact form first, and then also plug your answer(s) into your calculator and round to two digits after the decimal point.

9. (10 points) Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx} = 1 + y^2 + x^2 + y^2x^2, \qquad y(0) = 1.$$

10. (10 points) Scientists studying rat infestation in a certain big city believe that the carrying capacity of this human-made ecological niche is 10 million rats. They use the equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{10}\right)$$

to model the growth of the rat population, where t is time in years and P is the population of rats in millions. For example, $P = 3$ corresponds to a population of 3 million rats. The constant k characterizes the breeding conditions in this particular city.

Observations indicate that the rat population increased from 5 million in 2003 to 6 million in 2008. When should one expect the rat population to reach 9 million?

Give your answer as a date, rounded to the nearest year (for example, 2012).