Math 125 **Final Examination** **Spring 2005**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes. Do not share notes.
- Give your answers in exact form, except as noted in particular problems.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	20	
2	10	
3	10	
4	10	
5	10	

Question	Points	Score
6	10	
7	10	
8	10	
9	10	
Total	100	

1. (20 total points) Evaluate the following integrals. Simplify your answers where possible.

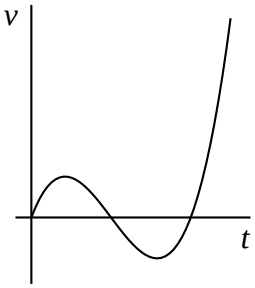
   (a) (5 points) $\int \frac{x^2}{\sqrt{x+2}}\,dx$

   (b) (5 points) $\int \tan^3(3t-1)\,dt$

(c) (5 points) $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

(d) (5 points) $\displaystyle\int_0^\infty \frac{1}{x^2+3x+2}\,dx$

2. (10 total points) A helicopter flies straight up and down from $t = 0$ to $t = 5$ seconds. Its velocity is $v(t) = t^3 - 6t^2 + 8t$ m/s; see the graph.



(a) (5 points) When does the helicopter return to its initial position? Justify your answer.

(b) (5 points) What total distance does the helicopter travel before returning to its initial position?

3. (10 total points)

   (a) (5 points) Evaluate $\int_{\pi/3}^{\pi/2} \frac{d}{dt}\Big(\sin(t)\cos(t/2)\Big)\,dt$

   (b) (5 points) Evaluate $\frac{d}{dx}\int_{e^x}^{\pi} \sin(t)\cos(t/2)\,dt$

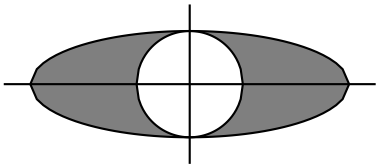
4. (10 points) Integrate the integral $\int_1^3 \ln(\ln(2z))\,dz$ numerically using Simpson's rule and six intervals. *Do not use your calculator* and *do not sum or otherwise simplify* your result.

5. (10 total points) Let R be the region in the first quadrant bounded by the curve $x = 1 - y^2$, the x-axis, and the y-axis. Let S be the solid obtained by rotating R about the line $x = -2$.

   (a) (5 points) Set up an integral for the volume of S using disks. DO NOT SIMPLIFY OR EVALUATE.

   (b) (5 points) Set up an integral for the volume of S using cylindrical shells. DO NOT SIMPLIFY OR EVALUATE.

6. (10 points) Find the area between the ellipse $\frac{x^2}{9}+y^2=1$ and the circle $x^2+y^2=1$.



7. (10 points) Find the length of the curve $y=\frac{1}{4}e^x+e^{-x}$, $0\le x\le 1$.

8. (10 total points) A culture of bacteria is growing in 10 liters of medium containing the sugar glucose. The bacteria consume 0.01 grams of glucose per minute from each liter of medium. Fresh medium containing 5g/L glucose is introduced to the culture at a rate of 2L/minute. Used medium is removed from the culture at the same rate.

   Assume that new medium mixes instantaneously with the existing medium as it is introduced.

   Let $x(t)$ denote the total amount of glucose in the culture at time t.

   (a) (5 points) Write down the differential equation that governs $x(t)$.

   (b) (5 points) Let the *concentration* of glucose in the medium at $t = 0$ be equal to 5g/L. Solve the differential equation.

9. (10 points) An aquarium plans to build an underwater gallery with transparent walls traversing the bottom of a large tank so that visitors can better view the fish. The gallery will have length L and will have a triangular cross section of height h and width $2r$. The height of the water in the tank measured from the base of the gallery will be H ($H \geq h$). Let ρ denote the density of water and let g be the acceleration due to gravity. What total force acts on each of the angled walls of the gallery?