**Math 125** **Final Examination** **Spring 2006**

Your Name | Your Signature

Student ID # | Quiz Section

Professor's Name | TA's Name

- This exam is closed book. You may use one $8\frac{1}{2}$" $\times$ 11" sheet of handwritten notes (both sides). Do not share notes.
- Give your answers in exact form, except as noted in particular problems.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may use any of the 20 integrals from the table on p. 506 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

| Question | Points | Score |
|---|---|---|
| 1 | 14 | |
| 2 | 14 | |
| 3 | 8 | |
| 4 | 8 | |
| 5 | 8 | |
| 6 | 8 | |

| Question | Points | Score |
|---|---|---|
| 7 | 8 | |
| 8 | 8 | |
| 9 | 8 | |
| 10 | 8 | |
| 11 | 8 | |
| Total | 100 | |

1. (14 total points) Evaluate the following integrals.

   (a) (7 points) $\displaystyle\int \frac{x}{\sqrt{8-2x-x^2}}\,dx$

   (b) (7 points) $\displaystyle\int \ln(\sec(x))\sec(x)\tan(x)\,dx$

2. (14 total points) Evaluate the following integrals. Leave your answers in exact form: do not use decimal expansions.

   (a) (7 points) $\int_0^{\pi/4} \sin^3(2x)\cos^2(2x)\,dx$

   (b) (7 points) $\int \frac{\cos x}{4-\sin^2 x}\,dx$

3. (8 points) Evaluate the integral

$$\int_2^\infty x^5 e^{-x^3}\,dx.$$

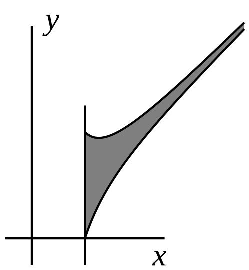
4. (8 points) A particle is moving along a straight line with acceleration $a(t) = 2t$. At time $t = 0$, its velocity is $v_0 = -4$. What is the *total distance* traveled by the particle from time $t = 0$ to time $t = 3$?

5. (8 points) Approximate the integral $\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-x^2/2}\, dx$ by using the trapezoidal rule with $n = 4$. Express your answer as a decimal.

6. (8 points) Consider the *unbounded* region $S$ contained within the curves

$$y = x + \frac{1}{x^2}, \qquad y = x - \frac{1}{x^2} \qquad \text{and} \qquad x = 1$$

as shown in the picture below.

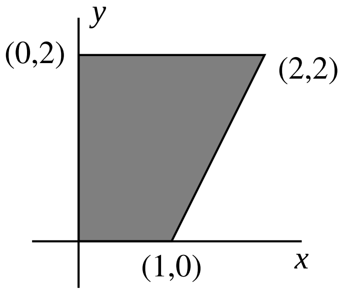



Is the area of $S$ finite or infinite? If it is finite, justify your conclusion and find this area. If it is infinite, carefully explain why.

7. (8 points) Let $R$ be the region below the curve $y = \frac{1}{x}$, above the $x$-axis, and between the vertical lines $x = 1$ and $x = 3$. Set up and evaluate a definite integral for the volume of the solid obtained by rotating $R$ about the vertical line $x = -2$.

8. (8 points) A small circular pool has a radius of 10 ft, the sides are 3 ft high, and the depth of the water is 2 ft. How much work (in ft-lb) is required to pump all of the water out over the side of the pool? (Water weighs 62.5 lb/ft$^3$.)

9. (8 points) Find the $x$-coordinate $\bar{x}$ of the center of mass of the region below.



10. (8 points) Find the solution $y(x)$ for $x \geq 1$ of the initial value problem

$$\frac{y}{x^3}\frac{dy}{dx} = 4\ln(x) \quad , \quad y(1) = 2.$$

11. (8 total points) Suppose we have a colony of bacteria living in a Petri dish. Due to space limitations, there is a maximum number, $k$, of bacteria that can live in the dish. Let $P(t)$ be the population of the bacterial colony at time $t$. According to one model for population growth, the rate of growth of the population $\frac{dP}{dt}$ is proportional to the difference of the threshold population $k$ and the present population; in other words

$$\frac{dP}{dt} = c(k-P). \tag{1}$$

The constant of proportionality $c$ measures how quickly the bacteria multiply. For simplicity, we take $c = 1$.

(a) (6 points) Solve this differential equation for the unknown function $P(t)$.

(b) (2 points) If $k = 5,000,000$ and the initial population size is $P(0) = 1,000,000$, compute $\lim_{t\to\infty} P(t)$.