Math 125 **Final Examination** **Winter 2003**

Your Name	Your Signature

Student ID #							Quiz Section	

Professor's Name	TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Give your answers in exact form. Do not give decimal approximations.
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- Place a box around YOUR FINAL ANSWER to each question.
- If you use a trial and error (or guess and check) method when an algebraic method is available, you will not receive full credit.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	15	
2	15	
3	6	
4	8	
5	6	
6	8	

Problem	Total Points	Score
7	10	
8	12	
9	8	
10	12	
Total	100	

1. [**15 points total**] Evaluate the following indefinite integrals. Justify your answers.

(a) [**5 points**] $\displaystyle\int \frac{2e^t}{1-e^{2t}}\,dt$

(b) [**5 points**] $\displaystyle\int \frac{t+7}{\sqrt{5-t}}\,dt$

(c) [**5 points**] $\displaystyle\int x\sec^3\left(x^2\right)\tan\left(x^2\right)\,dx$

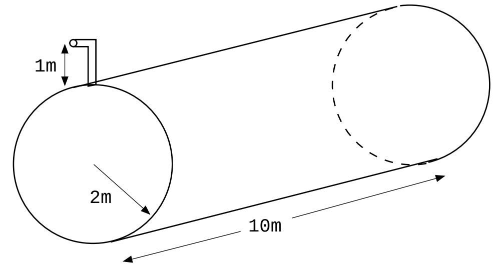
2. [**15 points total**] Evaluate the following definite integrals. Justify your answers.

(a) [**5 points**] $\int_0^{\pi/2} \cos^5 x \, dx$.

(b) [**5 points**] $\int_{-1}^{1} x \, e^{x^4} \, dx$.

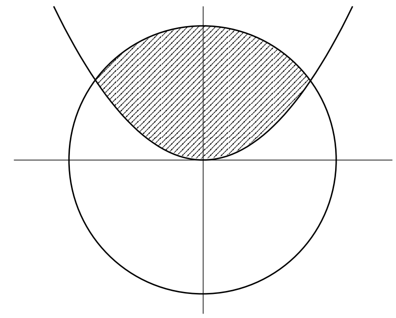
(c) [**5 points**] $\int_0^{\pi} x \sin(x/2) \, dx$

3. [**6 points**] A cylindrical tank is full of water (see the figure). Set up a definite integral for the work required to pump the water out of the outlet. Recall that the density of water is 1000 kg/m^3 and that the acceleration due to gravity is 9.8 m/sec^2. SET UP ONLY. DO NOT EVALUATE THE INTEGRAL.



4. [**8 points**] Use Simpson's rule with $n = 4$ subintervals to approximate the length of the curve $y = \sqrt{x}$, for $1 \leq x \leq 2$.

5. [**6 points**] Set up a definite integral for the area of the region (shown in the figure) enclosed by the curves $x^2 + y^2 = 6$ and $y = x^2$. SET UP ONLY. DO NOT EVALUATE THE INTEGRAL.



6. [**8 points**] Consider the integral $\int_0^6 \frac{1}{(x-4)^{2/3}}\,dx$. Determine whether this integral has a finite value (**carefully justify your answer**) and if so, find that value.

7. [**10 points total**] Consider the region R in the plane bounded by the curves

$$y = 4x - x^2 \qquad \text{and} \qquad y = x.$$

(a) [**5 points**] Sketch the region R and find its area.

(b) [**5 points**] Find the x-coordinate of the centroid of R.

8. [**12 points total**] A population of bacteria is observed to multiply in such a way that, if left alone, it would grow exponentially, doubling every $3\ln 2$ hours.

(a) [**4 points**] Write down a differential equation satisfied by the population $P(t)$ of bacteria if they are left alone (with t in hours). *Be sure to evaluate (exactly) any constant(s) in your differential equation.* DO NOT SOLVE THE EQUATION.

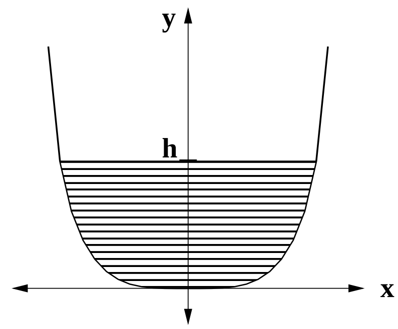
(b) [**3 points**] An antibiotic is introduced that kills bacteria at a constant rate of 1,000,000 bacteria per hour. Write down a differential equation satisfied by the population $P(t)$ in this case. DO NOT SOLVE THE EQUATION, YET.

Problem 8 continues on the next page...

(c) [**5 points**] Suppose that there are 2,000,000 bacteria present at time $t = 0$ hours. Solve the differential equation in part (b) to find a formula for $P(t)$. How long does it take until all of the bacteria are gone? *Give an EXACT answer, not a decimal.*

9. [**8 points**] Find **all** solutions of $y' = \dfrac{xe^x}{y}$.

10. [**12 points total**] The graph of $y = \tan\left(\frac{\pi}{3}x^4\right)$ between $x = 0$ and $x = 1$ is rotated around the y-axis to form a container that holds water. Units of length are in meters. Water is being poured into the container.



(a) [**4 points**] Let $V(h)$ be the volume of water in the container when the the water level is h meters above the x-axis. Express $V(h)$ as a definite integral with respect to y. DO NOT EVALUATE THE INTEGRAL.

(b) [**4 points**] Compute $\frac{dV}{dh}$ as a function of h.

(c) [**4 points**] Suppose that water is poured into the container at a constant rate of $\frac{dV}{dt} = 2$ cubic meters per second. Find $\frac{dh}{dt}$, the rate at which the water level is increasing, when $h = 1$ meter.