

Your Name

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Your Signature

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Student ID #

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Quiz Section

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Professor's Name

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TA's Name

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- This exam is closed book. You may use one  $8\frac{1}{2}'' \times 11''$  sheet of handwritten notes (both sides). Do not share notes.
- Give your answers in exact form, except as noted in particular problems.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place 

|                          |
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| a box around your answer |
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 to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

| Question | Points | Score |
|----------|--------|-------|
| 1        | 14     |       |
| 2        | 14     |       |
| 3        | 6      |       |
| 4        | 10     |       |
| 5        | 10     |       |
| 6        | 10     |       |

| Question | Points | Score |
|----------|--------|-------|
| 7        | 8      |       |
| 8        | 10     |       |
| 9        | 8      |       |
| 10       | 10     |       |
| Total    | 100    |       |

1. (14 total points) Evaluate the following integrals.

(a) (7 points)  $\int (\sin x)^3 dx$

(b) (7 points)  $\int \frac{x^3 + 2}{x^2 - 1} dx$

2. (14 total points) Evaluate the following integrals.

(a) (7 points)  $\int_0^{\pi/4} \tan \theta e^{2 \tan \theta} \sec^2 \theta d\theta$

(b) (7 points)  $\int_2^{7/2} \frac{x+1}{\sqrt{5+4x-x^2}} dx$

3. (6 points) Evaluate the improper integral

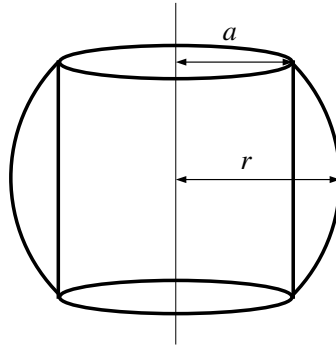
$$\int_0^4 \frac{1}{(x-3)^2} dx$$

or explain why it does not converge.

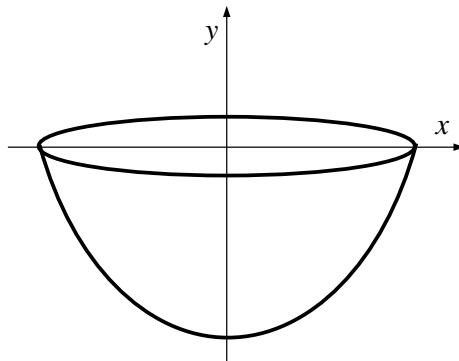
4. (10 total points) A small electric car travels along a straight track. At time  $t$  seconds, its velocity is given by  $v(t) = 12t - 3t^2$  ft/sec. The car starts moving at time 0.
- (a) (5 points) Find the distance of the car from its starting point after 5 seconds.

- (b) (5 points) Find the *total distance* traveled by the car during the first 5 seconds.

5. (10 points) A cylindrical hole of radius  $a$  is drilled through the center of a sphere of radius  $r$ , where  $a < r$ . Find the volume of the portion of the sphere that remains.



6. (10 points) A tank full of water is on the surface of Mars, where the gravitational acceleration is  $3.7\text{m/sec}^2$ . As shown below, the tank is in the shape of the curve  $y = \frac{1}{3}x^2 - 3$  (the part below the  $x$ -axis) rotated around the  $y$ -axis; the units are meters. Find the work required to pump the water out of an outlet at the top of the tank, which is at the level of the  $x$ -axis. Recall that the density of water is  $1000\text{ kg/m}^3$ . Give your answer in decimal form.



7. (8 total points)

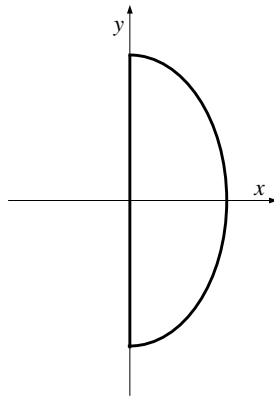
- (a) (4 points) Set up a definite integral for the arclength of the curve  $y = \sin(2x)$  for  $0 \leq x \leq 2\pi/3$ . Simplify the integrand but DO NOT EVALUATE THE INTEGRAL.

- (b) (4 points) Give an approximate value for the arclength by using the Trapezoid Rule with  $n = 4$  subintervals to approximate the integral in part (a). Give your answer in exact form.

8. (10 points) The half ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1, \quad x \geq 0,$$

is shown below. Its area is  $3\pi$ . Find its centroid.



9. (8 points) Find the solution of the differential equation

$$\frac{dy}{dx} = (x+1)(y+2)$$

which satisfies the initial condition  $y(0) = 7$ .

10. (10 total points) At time  $t = 0$  a 5000 liter tank is full of pure water. Starting at that moment salt is added to it at the steady rate of 30 grams per hour. Assume that the salt is thoroughly mixed in the water. Meanwhile, pure water is entering the tank at 50 liters per hour and the salty water in the tank is leaving at the same rate. Let  $y(t)$  be the amount of salt in grams in the water in the tank after  $t$  hours.
- (a) (4 points) Find a differential equation for  $y(t)$ .
  - (b) (4 points) Solve for  $y(t)$ .
  - (c) (2 points) How much salt is in the tank in the limit as  $t \rightarrow \infty$ ?