Math 125 **Final Examination** **Winter 2008**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2}" \times 11"$ sheet of handwritten notes (both sides). Do not share notes.
- Give your answers in exact form, except as noted in particular problems.
- Graphing calculators are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. You may use any of the 20 integrals on p. 506 of the text without deriving them. Show your work in evaluating any other integrals, even if they are on your note sheet.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 10 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	12	
2	12	
3	6	
4	12	
5	10	
6	10	

Question	Points	Score
7	8	
8	10	
9	8	
10	12	
Total	100	

1. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\int x^3 \cos(x^2)\, dx$

   (b) (6 points) $\int \frac{(\sqrt{\ln(x)}+1)^3}{x\sqrt{\ln(x)}}\, dx$

2. (12 total points) Evaluate the following integrals.

   (a) (6 points) $\displaystyle\int \frac{2x-1}{x^2-3x+2}\,dx$

   (b) (6 points) $\displaystyle\int \frac{\sqrt{3-2x-x^2}}{x+1}\,dx$

3. (6 points) Consider the improper integral

$$\int_1^\infty \frac{\ln(x)}{x^2}\,dx.$$

Evaluate this integral or explain why it does not converge.

4. (12 total points) A particle moves along a straight line with acceleration (in $\mathrm{m}/\mathrm{sec}^2$)

$$a(t) = e^{3-t}.$$

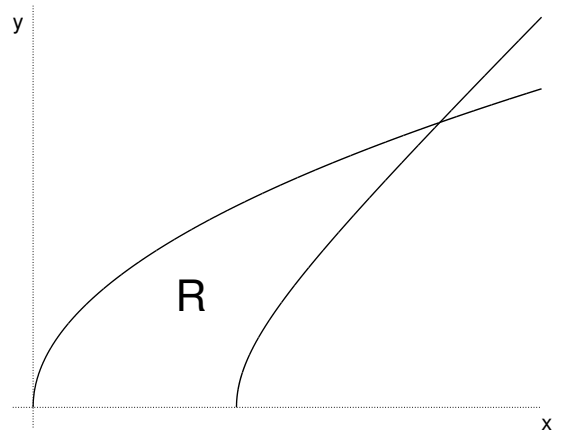
At time $t = 0$, the velocity of the particle is $1 - e^3$.

(a) (4 points) Find the velocity function $v(t)$ of the particle.

(b) (4 points) Find the displacement of the particle from $t = 0$ to $t = 5$.

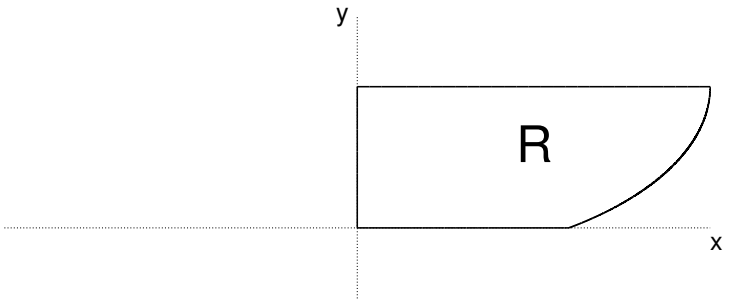
(c) (4 points) Find the *total distance* traveled by the particle during the time interval $0 \leq t \leq 5$.

5. (10 points) The region R as shown below is bounded by the curves $y = \sqrt{x}$, $y = \sqrt{x(x-1)}$, and the x-axis.



Find the volume of the solid formed by rotating R around the x-axis.

6. (10 points) The region R bounded by the y-axis, the lines $y = 4$ and $y = 0$, and the portion of the ellipse $x^2 + 4(y-4)^2 = 100$ between the points (6,0) and (10,4) is revolved around the y-axis to form a container that is full of water. (See picture below.)



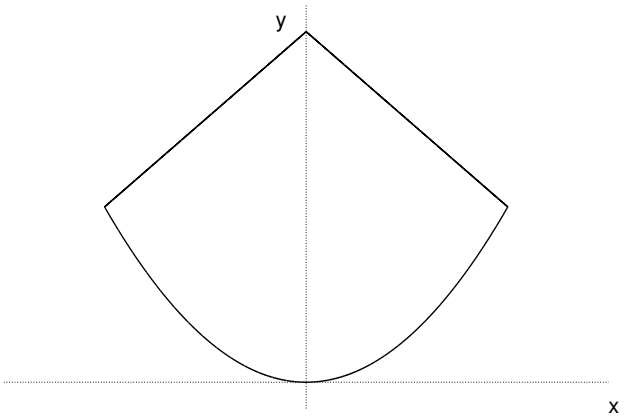
The units are feet, and the density of water is $62.5\ \text{lb}/\text{ft}^3$. Find the work needed to empty the container; that is, to lift all the water up to the level of the top of the container.

7. (8 total points)

   (a) (4 points) Set up a definite integral for the arclength of the curve $y = \ln(x)$ for $1 \leq x \leq 3$. **Do not evaluate the integral.**

   (b) (4 points) Give an approximate value for the arclength by using Simpson's rule with $n = 4$ subintervals to approximate the integral in part (a). Make sure to show your work clearly.

8. (10 points) Find the centroid of the region shown below. It is bounded by the lines $y = x + 4$, $y = -x + 4$, and the curve $y = x^2/2$.



9. (8 points) Find the solution of the differential equation

$$\frac{dy}{dx} = x^2 y \sec^2(x^3)$$

which satisfies the initial condition $y(0) = -3$.

10. (12 total points) A tank contains 100 liters of water which has 2000 grams of salt dissolved in it. At noon, pure water begins to enter the tank at the rate of 10 liters per minute. The tank is kept thoroughly mixed, and the mixture leaves the tank at the rate of 15 liters per minute.

    (a) (2 points) Express the volume of saltwater in the tank as a function of t, the number of minutes after noon.

    (b) (4 points) Let $S(t)$ be the amount of salt in grams in the tank at time t. Set up a differential equation describing $\dfrac{dS}{dt}$.

    (c) (4 points) Solve the differential equation for $S(t)$.

    (d) (2 points) When will there be 1000 grams of salt in the tank?