Math 125, Sections C and F, Fall 2014, Midterm I

October 16, 2014

Name Solutions

TA/Section

Instructions.

- There are 4 questions. The exam is out of 40 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. Hand in you note sheet with your exam.
- Calculators are NOT allowed. Put away ALL electronic devices.
- For your integrals you may use the following formulas. Anything else must be justified by your work.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \qquad \int e^x\,dx = e^x + C \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C \qquad \int \sec^2 x\,dx = \tan x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C \qquad \int \sec x \tan x\,dx = \sec x + C = \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2}dx = \tan^{-1} x + C$$

- **Show your work**. If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note for me.

Question	points
1	
2	
3	
4	
Total	

1. (10 points) Evaluate the following integrals.

(a) $\int 7\cos(\theta)\sin^2(\theta)d\theta$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$\int 7u^2 d\theta = \frac{7}{3}u^3 + C = \frac{7}{3}\sin^3\theta + C$

(b) $\int_0^1 \frac{x}{1+5x}dx$

$u = 1+5x \longrightarrow x = \frac{u-1}{5}$

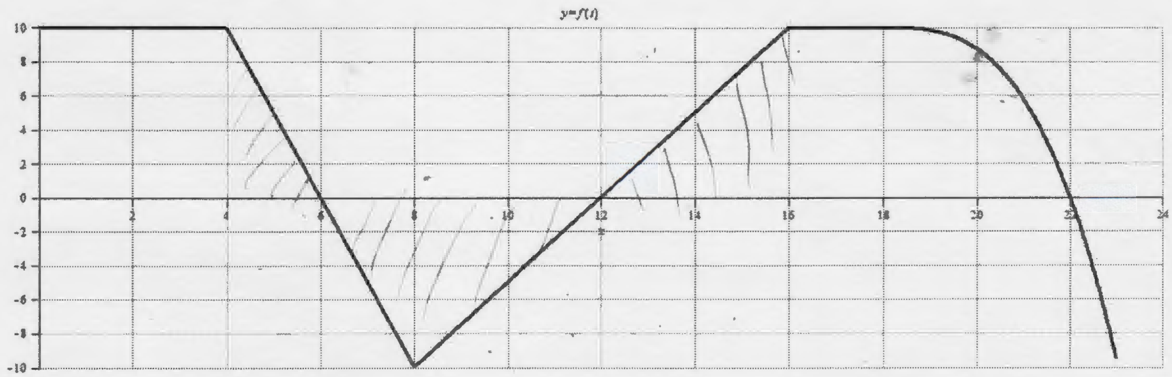
$du = 5\,dx$

$= \int_1^6 \frac{\frac{u-1}{5}}{u} \frac{1}{5} du = \frac{1}{25}\int_1^6 \frac{u-1}{u} du = \frac{1}{25}\int_1^6 1 - \frac{1}{u} du$

$= \frac{1}{25}\left[u - \ln|u|\right]_1^6 = \frac{1}{25}(5 - \ln 6)$

(c) $\int_{-1}^1 xe^{x^8}dx = 0$ because xe^{x^8} is an odd function.

2. (10 points) Define $g(x) = \int_4^x f(t)dt$ where the graph of $f(t)$ is given below.



(a) Evaluate the following:

$g(0) = \int_4^0 f(t)dt = -\int_0^4 f(t)dt = -40$

$g(4) = 0$

$g(16) = \frac{2\cdot 10}{2} - \frac{6 \cdot 10}{2} + \frac{4 \cdot 10}{2} = 10 - 30 + 20 = 0$

$g'(17) = f(17) = 10$

$g''(11) = f'(11) = \frac{20}{8} = 2.5$

(b) Express $g(22) - g(18)$ as a definite integral and estimate it with $n = 4$ and leftpoints. This question will be graded with a reasonable allowance for estimation error.

$\Delta t = \frac{22-18}{4} = 1$

$\int_{18}^{22} f(t)dt \approx [f(18) + f(19) + f(20) + f(21)]\Delta t$

$\approx 10 + 9.8 + 8.8 + 5.8$

≈ 34.4

(c) If $h(x) = \int_4^{x^3} f(t)dt$, what is $h'(2)$?

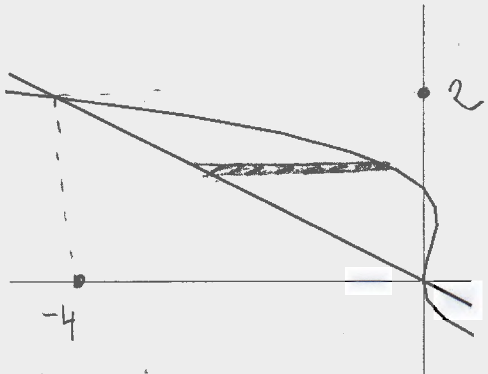
$h(x) = g(x^3)$

$h'(x) = g'(x^3) \cdot 3x^2$

$= f(x^3) \cdot 3x^2$

$h'(2) = f(8) \cdot 3 \cdot 4 =$ $\boxed{(-10)(3)(4) = -120}$

3. Find the area of the region shown below bounded by the curve $x = -y^3 + y^2$ and the line $x = -2y$.



Points of intersection:

$-y^3+y^2=-2y$

$-y^3+y^2+2y=0$

$-y(y^2-y-2)=0$

$-y(y-2)(y+1)=0$

$y=0 \quad y=2 \quad y=-1$

$x=0 \quad x=-4 \quad x=2$

$$A=\int_0^2 (-y^3+y^2)-(-2y)\,dy$$

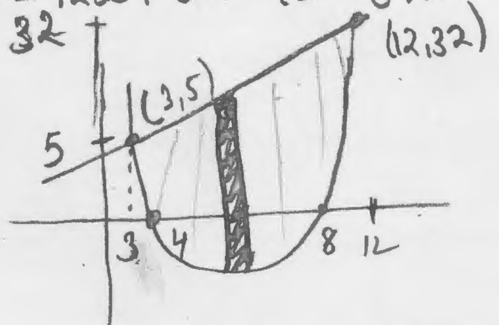
$$=\int_0^2 -y^3+y^2+2y\,dy = -\frac{y^4}{4}+\frac{y^3}{3}+y^2\Big|_0^2$$

$$=-4+\frac{8}{3}+4=\frac{8}{3}.$$

4. (11 points)

(a) Sketch the region between the parabola $y = x^2 - 12x + 32$ and the line $y = 3x - 4$. Label all intersection points.

$y = x^2 - 12x + 32 = (x-8)(x-4)$



$3x - 4 = x^2 - 12x + 32$

$0 = x^2 - 15x + 36$

$0 = (x-12)(x-3)$

$x = 12 \quad x = 3$

$y = 32 \quad y = 5$

$x - 1$

$y = (x-9)(x-5)$

$x^2 - 14x + 45$

$y = 3(x-1) - 4$

$= 3x - 7$

(b) Set up an integral to calculate the volume of the solid formed by rotating this region about the y-axis. Do NOT integrate.

$$\int_3^{12} 2\pi x \left[3x - 4 - (x^2 - 12x + 32)\right] dx$$

(c) Set up an integral to calculate the volume of the solid formed by rotating this region about the horizontal line $y = 40$. Do NOT integrate.

$$\int_3^{12} \pi \left[\left(40 - (x^2 - 12x + 32)\right)^2 - \left(40 - (3x - 4)\right)^2 \right] dx$$

(d) Set up an integral to calculate the volume of the solid formed by rotating this region about the vertical line $x = 4$. Do NOT integrate.

This is rotating the part to the left of $x=4$ about $x=4$

$$\int_4^{12} 2\pi (x-4)\left(3x - 4 - (x^2 - 12x + 32)\right) dx.$$