Name____________________________________

Math 125 **First Midterm** **11:30-12:50, Oct. 19, 2017**

(5 problems, 80 minutes, 100 points; 1 sheet of notes but no calculator, no cellphone, no watch permitted)

1. (30 points) Find the indefinite integrals in (a) and (b) and the definite integral in (c). In (c) give your answer in exact form, that is, in terms of π, e, and squareroots. In all three parts, please show your work clearly, and make any obvious simplifications.

(a) $$\int \frac{100x\,dx}{(5x+6)^{2/3}}$$

(PARTS (b) AND (c) ON NEXT PAGE)

(b) $\displaystyle\int \frac{x^3\,dx}{1+x^8}$

(c) $\displaystyle\int_0^{\pi} e^{\tan(x/3)}\sec^2(x/3)dx$

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2. (15 points) A ball is dropped from a window located at 80 meters above the ground. Assume that the gravitational acceleration of the ball is -10 m/sec^2. Please show your work clearly.

(a) (6 points) Suppose that the ball is released with no initial vertical velocity. Find the time of the impact and the vertical velocity of the ball at the time of impact. Please show your work clearly.

(b) (9 points) What initial vertical velocity should be given to the ball so that the ball hits the ground in 3/4 the time computed in part (a)?

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3. (15 points) Let $T(t)$ denote the temperature in degrees Celsius at time t, where t is measured in **days**. Suppose that this is during the winter, when $T(t)$ is always less than the comfort level 18°C. By definition, the number of degree-days during the time interval from $t = a$ to $t = b$ is the area between the constant function $y = 18$°C and the temperature function $y = T(t)$ between $t = a$ and $t = b$. The number of degree-days is useful in estimating the cost of heating a building to 18°C during this period. Suppose that during a 3-day period temperature readings are taken at 6-hour intervals, starting at $t = 0$, which is midnight at the beginning of the first day. Let T_0, T_1, $T_2, \ldots$, T_{12} denote the 13 readings taken between the beginning of the first day and the end of the third day. Use the trapezoid rule to write an expression involving $T_0, \ldots, T_{12}$ for the number of degree-days over the three-day period.

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4. (20 points) (a) Write

$$\lim_{n\longrightarrow\infty} \frac{2}{n} \sum_{i=1}^{n} 2\pi \left(\frac{2i}{n}\right) \frac{1}{3+\left(\frac{2i}{n}\right)^2}$$

as a definite integral.

(b) Explain what volume is given by the integral in part (a), by filling in the blanks:

This is the volume obtained by rotating the region R

around ____________________, where R is the region

between the graph of the function ________________ and

the _____$-$axis between ___ $=$ ___ and ___ $=$ ___.

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5. (20 points) Let R be the region bounded by the two curves $y = x^3 - x^2$ and $y = 2x^2$. Write each of the following in terms of definite integrals (being careful to indicate the limits of integration). Do **not** evaluate the integrals.

(a) The volume when R is revolved around the line $y = -4$.

(b) The volume when R is revolved around the line $x = -1$.

1. (a) Set $u = 5x + 6$, so that $x = (u - 6)/5$ and $dx = \frac{1}{5}du$, so you get $\frac{100}{25}\int \frac{u-6}{u^{2/3}}du$ $=4\int(u^{1/3} - 6u^{-2/3})du$$=4(\frac{3}{4}u^{4/3} - 6 \cdot 3u^{1/3}) + C$$=$
$=3(5x + 6)^{4/3} - 72(5x + 6)^{1/3} + C$.

(b) Set $u = x^4$, so that $x^8 = u^2$ and $du = 4x^3dx$, so that our integral becomes $\frac{1}{4}\int \frac{du}{1+u^2}$ $=\frac{1}{4}\text{Arctan}(u) + C$$=\frac{1}{4}\text{Arctan}(x^4) + C$.

(c) Set $u = \tan(x/3)$, so that $du = \frac{1}{3}\sec^2(x/3)dx$ and the limits of integration become 0 to $\sqrt{3}$ (because $\tan(\pi/3) = \sqrt{3}$). We then get
$3\int_0^{\sqrt{3}} e^u du = 3e^u \Big|_0^{\sqrt{3}} = 3(e^{\sqrt{3}} - 1)$.

2. This problem is very similar to the last problem on the Burdzy practice midterm. (a) Solve $-5t^2 + 80 = 0$ for t to get $t = 4$ sec. Since $v = -10t$, at impact we have $v = -40$ m/sec (that is, 40 m/sec in the downward direction). (b) With nonzero v_0 we have the formula $-5t^2 + v_0t + 80$, which must equal 0 (that is, the ball is on the ground) when $t = 3$ sec. Hence $-5 \cdot 3^2 + 3v_0 + 80 = 0$, and solving this for v_0 gives a downward velocity of $\frac{35}{3} = 11\frac{2}{3}$ m/sec.

3. $\frac{1}{4}(\frac{1}{2}(18 - T_0) + (18 - T_1) + (18 - T_2) + \cdots + (18 - T_{11}) + \frac{1}{2}(18 - T_{12}))$, or, equivalently, $54 - \frac{1}{4}(\frac{1}{2}T_0 + T_1 + T_2 + \cdots + T_{11} + \frac{1}{2}T_{12})$.

4. This problem is similar to #5 on the practice midterm that was passed out last week.
(a) $\int_0^2 2\pi x \frac{1}{3+x^2}dx$.
(b) This is the volume obtained by rotating the region R around the y-axis, where R is the region below the graph of the function $(3+x^2)^{-1}$ and above the x-axis between $x = 0$ and $x = 2$.

5. The curves intersect when $x^3 - x^2 = 2x^2$, that is, when $x^2(x - 3) = 0$; this is at $(0, 0)$ and at $(3, 18)$. The curve $y = 2x^2$ is on top.
(a) $\pi\int_0^3 \left((2x^2 + 4)^2 - (x^3 - x^2 + 4)^2\right) dx$.
(b) $2\pi\int_0^3(x + 1)(2x^2 - (x^3 - x^2))dx = 2\pi\int_0^3(x + 1)(3x^2 - x^3)dx$.