# MIDTERM #1 – Answers

## Math 125H

name TA section

You must show all work for full credit. Use the backs of the test pages as necessary. Give all answers in EXACT FORM (no decimal approximations).

1. Evaluate the following integrals.

a. $\displaystyle\int_1^4 \frac{4}{x+2}\,dx = 4\ln(x+2)\Big|_1^4 = 4\ln(6) - 4\ln(3) = \ln(16)$

b. $\displaystyle\int_\pi^{2\pi} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = \int_{\sqrt{\pi}}^{\sqrt{2\pi}} 2\cos(u)\,du = 2\left(\sin\sqrt{2\pi} - \sin\sqrt{\pi}\right)$

c. $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\,dx = \frac{\pi}{2}$ (area of a unit semicircle)

2. Evaluate

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1}\sqrt{\frac{i}{n}}$$

by recognizing this limit as a Riemann sum for a certain integral.

This is a (left) Riemann sum for $\displaystyle\int_0^1 \sqrt{x}\,dx$, so the value is $\displaystyle\frac{2}{3}x^{3/2}\Big|_0^1 = \frac{2}{3}$.

3. Using the Fundamental Theorem of Calculus, evaluate

$$\lim_{x\to 5} \frac{1}{x-5}\int_5^x \ln t\,dt.$$

This is $F'(5)$, where $F(x) = \displaystyle\int_5^x \ln(t)\,dt$, so it is $\ln 5$.

4. Find the volume of the solid of revolution obtained by rotating the region under the graph of $y = 1/x$ between the limits $x = 1$ and $x = 2$ about the $x$-axis.

$$\text{Volume} = \int_1^2 \pi\left(\frac{1}{x}\right)^2 dx = -\frac{\pi}{x}\Big|_1^2 = \frac{\pi}{2}, \text{ by slices.}$$

5. Find the volume of the solid of revolution obtained by rotating a right triangle with vertices $(0,0), (2,2)$, and $(4,0)$ about the $y$-axis.

This is

$$\int_0^2 \pi\left[(4-y)^2 - y^2\right] dy = \int_0^2 16\pi - 8\pi y\,dy = 16\pi y - 4\pi y^2\Big|_0^2 = 16\pi$$

by slices. Or, by shells,

$$\begin{aligned}\int_0^2 2\pi x^2\,dx + \int_2^4 2\pi x(4-x)\,dx &= \frac{2\pi x^3}{3}\Big|_0^2 + 4\pi x^2\Big|_2^4 - \frac{2\pi x^3}{3}\Big|_2^4 \\ &= \pi\left(\frac{16}{3} + 48 - \frac{128}{3} + \frac{16}{3}\right) = 16\pi.\end{aligned}$$