1 (9 points) Evaluate the following integrals:

(a) (3 points) $\int_1^2 (1+x^2)x^3\,dx$.

Ans: $14\frac{1}{4}$.

(b) (3 points) $\int_1^2 \frac{1+x^2}{x^3}\,dx$.

Ans: $\frac{3}{8}+\ln(2)$.

(c) (3 points) $\int_1^2 \frac{x^3}{1+x^2}\,dx$.

Ans: $\frac{3}{2}-\frac{1}{2}\ln(\frac{5}{2})$.

2 (10 points) Evaluate the following integrals:

(a) (5 points) $\int \frac{\sin(\sqrt{x})\cos(\sqrt{x})}{\sqrt{x}}\,dx$.

Ans: $(\sin(\sqrt{x}))^2+C$.

(b) (5 points) $\int_2^9 f(x)\,dx$ if $\int_2^4 [f(x)+g(x)]\,dx = 11$, $\int_{-2}^3 f(x^2)x\,dx = 13$ and $\int_2^4 [3g(x)+2]\,dx = 7$.

Ans: SInce $7 = \int_2^4 [3g(x)+2]dx = 3\int_2^4 g(x)dx + \int_2^4 2dx = 3\int_2^4 g(x)dx + 4$ we get $\int_2^4 g(x)dx = \frac{7-4}{3} = 1$. Since $11 = \int_2^4 [f(x)+g(x)]dx = \int_2^4 f(x)dx + \int_2^4 g(x)dx = \int_2^4 f(x)dx + 1$ we get $\int_2^4 f(x)dx = 10$. Since $13 = \int_{-2}^3 f(x^2)xdx$ we can take $u = x^2$ so $\frac{1}{2}du = xdx$ and at $x = -2$ we have $u = 4$ while at $x = 3$ we have $u = 9$ to get $13 = \int_4^9 f(u)\frac{1}{2}du$ so that $26 = \int_4^9 f(x)dx$. Thus $\int_2^9 f(x)\,dx = 36$.

3 (10 points)

(a) (5 points) Find the limit $\lim_{n\to\infty}\sum_{i=1}^n \frac{999}{n}\frac{1}{1+\frac{999}{n}i}$ by writing it as a definite integral and solving the integral.

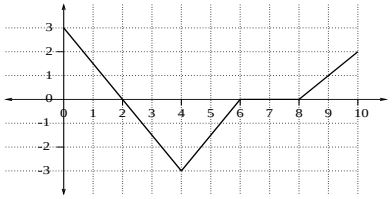
Ans: $\int_1^{1000}\frac{1}{x}dx = 3\ln(10)$.

(b) (5 points) Estimate the sum $\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{1000}$ by writing it as a right Riemann sum (R_{999}) and computing the integral for which R_{999} is an approximation.
Is your answer an over estimate or an under estimate for the sum?

Ans: $\int_1^{1000}\frac{1}{x}dx = 3\ln(10)$ is an overestimate.

4 (7 points) Consider the following graph of $f(t)$ as a function of t and define

$$g(x) = \int_0^x f(t)dt.$$



(a) (3 points) Find the values of $g(x)$ and $g'(x)$ at the even values of x between 0 and 10.

Ans:

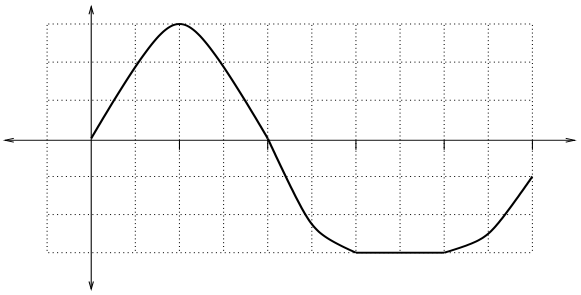
x	0	2	4	6	8	10
g(x)	0	3	0	-3	-3	-1
g'(x)	3	0	-3	0	0	2

(b) (2 points) Find the intervals on which g is increasing.

Ans: $(0, 2)$ and $(8, 10)$.

(c) (2 points) Sketch a graph of $g(x)$ as a function of x.

Ans:



(d) (Extra Credit): Find $h'(3)$ if $h(x) = \int_0^{x^2} f(t)\ dt$.

Ans: 6.

5 (14 points) Consider the region R bounded by the curves: $y = x^2$ and $y = x$.

(a) (4 points) Find the area of R.

Ans: $\frac{1}{6}$.

(b) (5 points) Find the volume of the solid formed by rotating R about the x-axis ($y = 0$).

Ans: $\frac{2\pi}{15}$.

(c) (5 points) Find the volume of the solid formed by rotating R about the line $x = -1$. (This is a vertical line.)

Ans: $\frac{\pi}{2}$.