

**Problem 1 (8 points)**

Find the function  $f(x)$  such that  $f''(x) = 3x + \cos(\pi x)$ , with  $f(0) = 0$ , and  $f'(0) = 3$ .

**Problem 2 (12 points).**

Evaluate the following integrals. (For (a), express your answer in exact form. For (b), find the most general anti-derivative.)

(a.)  $\int_2^3 \frac{[\ln(x)]^3}{x} dx$

(b.)  $\int \frac{\cos(1 + \sqrt{x})}{\sqrt{x}} dx$

**Problem 3 (9 points).**

Use the midpoint rule, with  $n = 5$  subintervals, to find an approximate value for the following definite integral. (Your answer should involve sines and square roots; do not convert things into decimals.)

$$\int_1^3 \sin(\sqrt{u}) \, du$$

**Problem 4 (9 points).**

The region between the curves  $y = x^2 - \frac{1}{2}x^4$  and  $y = -2x$  for  $0 \leq x \leq 2$  is rotated about the  $y$ -axis to form a solid of revolution. What is the volume of this solid?  
(Hint: use the cylindrical shell method.)

**Problem 5 (12 points).**

The curve  $y = 2x^4$ , for  $0 \leq x \leq 1$ , is rotated about the  $y$ -axis to form a container.

- a. How much liquid does this container hold when it is full (to height  $y = 2$ )?
- b. Find the value  $h$  so that, when the container is filled with liquid to height  $y = h$ , it is exactly one-half full. (That is, it contains one-half the amount of liquid you calculated in (a). Express the answer for  $h$  exactly, not in decimal form.)