Math 125I Midterm 1 Solutions Winter 2003

1.(a) $\displaystyle\int_1^8 \frac{2x+5}{\sqrt[3]{x^2}}\,dx = \int_1^8 2x^{1/3} + 5x^{-2/3}\,dx = \frac{3}{2}x^{4/3} + 15x^{1/3}\Big|_1^8 = \frac{75}{2}$

(b) Let $u = \cos t$ so $du = -\sin t\,dt$. Then $\displaystyle\int_0^{\pi} \frac{\sin t}{1+\cos^2 t}\,dt = -\int_1^{-1} \frac{1}{1+u^2}\,du = -\tan^{-1} t\Big|_1^{-1} = \frac{\pi}{2}$

(c) Let $v = y^2 - 7$ so that $dv = 2y\,dy$ and $y^2 = v + 7$. Then $\displaystyle\int y^3\sqrt{y^2-7}\,dy = \frac{1}{2}\int (v+7)\sqrt{v}\,dv =$ $\displaystyle\frac{1}{2}\int v^{3/2} + 7v^{1/2}\,dv = \frac{1}{5}v^{5/2} + \frac{7}{3}v^{3/2} + C = \frac{1}{5}(y^2-7)^{5/2} + \frac{7}{3}(y^2-7)^{3/2} + C$

2. $\Delta t = \frac{1}{2}$ and the t-coordinates of the midpoints are $\frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \frac{11}{4}, \frac{13}{4}, \frac{15}{4}$. The function $v(t)$ is positive at the first 4 values and negative at the last 2. Thus the total distance is

$$\frac{1}{2}\left[v\left(\frac{5}{4}\right) + v\left(\frac{7}{4}\right) + v\left(\frac{9}{4}\right) + v\left(\frac{11}{4}\right) - v\left(\frac{13}{4}\right) - v\left(\frac{15}{4}\right)\right] \approx 2.1784 \text{ feet.}$$

3. By the Fundamental Theorem of Calculus, $f'(x) = (2x-4)e^{\sqrt{x^2-4x}}$. This is defined when $x^2 - 4x \geq 0$, that is $x \geq 4$ or $x \leq 0$. Since $e^u > 0$ for any u, the derivative is positive if it is defined and if $2x - 4 > 0$. Thus the function is increasing when $x \geq 4$.

4. Solve $x^2 = x^3 - 6x^2 + 10x$ to get $x = 0, 2, 5$. Then compute

$$\int_0^2 \left(x^3 - 6x^2 + 10x\right) - \left(x^2\right)\,dx - \int_2^5 \left(x^3 - 6x^2 + 10x\right) - \left(x^2\right)\,dx = \frac{253}{12} \approx 21.083.$$

5. For both parts you need to solve $\dfrac{9}{x^2} = 13 - 4x$. This gives $4x^3 - 13x^2 + 9 = 0$ which is hard to solve by elementary methods. Guessing and checking, you find that $x = 1$ is a solution. Using long division, you get $4x^3 - 13x^2 + 9 = (x-1)(4x^2 - 9x - 9)$. The quadratic term has roots $x = 3, -\frac{3}{4}$. So the limits of integration are $x = 1$ to 3.

(a) $\displaystyle\pi\int_1^3 \left(13 - 4x\right)^2 - \left(\frac{9}{x^2}\right)^2\,dx$

(b) $\displaystyle 2\pi\int_1^3 \left(x+2\right)\left(13 - 4x - \frac{9}{x^2}\right)\,dx$