1. [**14 points**] Evaluate the following integrals. Show all steps. Simplify and box your answer.

(a) [**4 points**] $\int \frac{3x-2}{\sqrt{x}}\,dx$

$$= \int 3x^{1/2} - 2x^{-1/2}\,dx$$

$$= 3\,\frac{x^{3/2}}{3/2} - 2\,\frac{x^{1/2}}{1/2} + C$$

$$= \boxed{2x^{3/2} - 4x^{1/2} + C}$$

or: $\boxed{2(\sqrt{x})^3 - 4\sqrt{x} + C}$

(b) [**5 points**] $\int \sqrt{x}\sin(1+x^{\frac{3}{2}})\,dx$

U-SUB

$u = 1 + x^{3/2}$

$du = \frac{3}{2}x^{1/2}\,dx \Rightarrow \sqrt{x}\,dx = \frac{2}{3}\,du$

$$= \int \sin(u)\,\frac{2}{3}\,du$$

$$= \frac{2}{3}(-\cos u) + C$$

$$= \boxed{-\frac{2}{3}\cos(1+x^{3/2}) + C}$$

(c) [**5 points**] $\int_1^2 \frac{5}{2-3x}\,dx$

U-SUB

$u = 2-3x$

$du = -3\,dx$

Bounds: $x=1 \Rightarrow u=-1$

$x=2 \Rightarrow u=-4$

$$\int_1^2 \frac{5}{2-3x}\,dx = \int_{-1}^{-4} \frac{5}{u}\left(-\frac{1}{3}\right)du =$$

$$= -\frac{5}{3}\int_{-1}^{-4}\frac{1}{u}\,du = \frac{5}{3}\int_{-4}^{-1}\frac{1}{u}\,du$$

$$= \frac{5}{3}\ln|u|\Big|_{-4}^{-1} = \frac{5}{3}(\underbrace{\ln 1}_{=0} - \ln 4).$$

$$= \boxed{\frac{-5}{3}\ln 4}$$

2. [**7 points**]

A particle is moving along a straight line. At all times $t \geq 0$ the velocity of the particle is given by

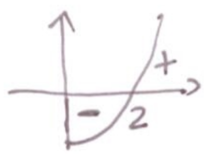
$$v(t) = 3t^2 - 12$$

Let b be an arbitrary number greater than 10. Find the total distance traveled by the particle from time $t = 0$ to time $t = b$. Your answer should be an expression involving b. Show all work.

Total distance $= \int_0^b |v(t)|\,dt = \int_0^b |3t^2 - 12|\,dt$

What is the sign of $v(t) = 3t^2 - 12$?

$v(t) = 0$ when $3t^2 - 12 = 0 \iff t^2 = \frac{12}{3} = 4 \iff t = \pm 2$



$v(t)$ is $\begin{cases} \text{negative for } 0 \leq t < 2 \\ \text{positive for } 2 < t \leq b \end{cases}$

$\to$ Total dist $= \int_0^2 -(3t^2 - 12)\,dt + \int_2^b (3t^2 - 12)\,dt$

$= (12t - t^3)\Big|_0^2 + (t^3 - 12t)\Big|_2^b$

$= (24 - 8) - (0 - 0) + (b^3 - 12b) - (8 - 24)$

$= 16 + b^3 - 12b + 16$

$= \boxed{b^3 - 12b + 32}$.

3. [**7 points**]

(a) You are given that $g(x)$ is a continuous function on [0,3] such that

$$\int_0^3 g(x)dx = -1 \text{ and } \int_2^3 g(x)dx = -3$$

$\Big\} \Rightarrow \int_0^2 = \int_0^3 - \int_2^3$
$= (-1)-(-3)$
$= 2$

Compute $\int_0^2 5g(x) + 7\, dx$. Show all steps.

$$\int_0^2 5g(x)+7\,dx = 5\underbrace{\int_0^2 g(x)\,dx} + \underbrace{\int_0^2 7\,dx}$$
$$= 5\ (2)\ + 7(2-0)$$
$$= 10 + 14$$
$$= \boxed{24}.$$

(b) Sue and Kathy race each other, running with continuous positive velocities $v_S(t)$ and $v_K(t)$, respectively. They start the race at the starting line at $t = 0$ seconds. Kathy runs faster than Sue throughout the race. Write a definite integral that would equal the area between their velocity curves over the first 10 seconds of the race, and a brief English sentence giving the physical interpretation of what that area and integral represent.

$$\text{Area} = \int_0^{10} V_K(t) - V_S(t)\,dt$$

This is the distance by which Kathy is ahead of Sue at the end of the 10 seconds.
(or: the distance between the runners at $t = 10$ sec)

4. [**8 points**] Compute each of the following expressions. Justify your answer.

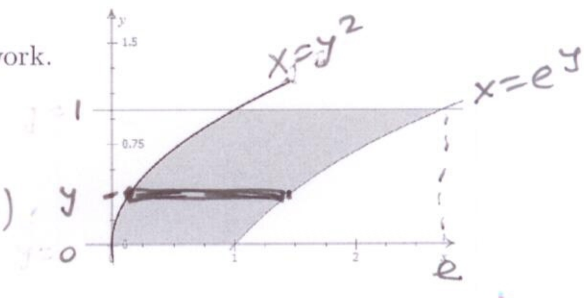
(a) $\frac{d}{dx}\int_0^{3x} \sin(t^2)\,dt \underset{\text{FTC I + Chain Rule}}{=} \sin((3x)^2)\cdot(3x)' = \boxed{\sin(9x^2)\cdot 3}$

(b) $\int_0^3 \frac{d}{dx}(\sin(x^2))\,dx = \sin(x^2)\Big|_0^3 = \sin(9) - \sin(0) = \boxed{\sin(9)}$

(c) $\frac{d}{dx}\int_0^3 \sin(t^2)\,dt = \frac{d}{dx}(\text{a number}) = \boxed{0}!$

5. **[14 points]** Let $\mathcal{R}$ denote the region bounded by the graphs of $x = y^2$, $x = e^y$, $y = 0$, and $y = 1$.

(a) **[6 points]** Compute the **area** of this region $\mathcal{R}$. Show your work.

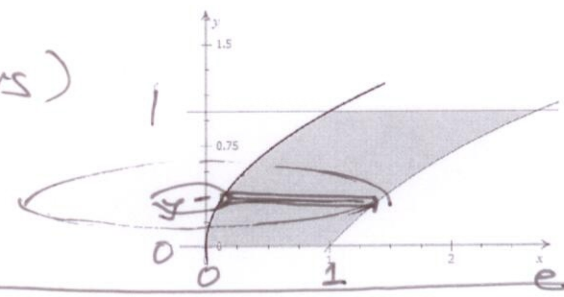


$$A = \int_0^1 (e^y - y^2)\,dy = e^y - \tfrac{1}{3}y^3 \Big|_0^1$$
$$= (e - \tfrac{1}{3}) - (1-0)$$
$$= \boxed{e - 4/3}$$

In x: $A = \int_0^1 \sqrt{x}\,dx + \int_1^e (1 - \ln x)\,dx$ (harder & we don't yet know how to integrate $\ln x$)

(b) **[8 points]** **SET UP** (but **DO NOT EVALUATE**) definite integrals equal to the **volumes** of the solids of revolution obtained by rotating the same region $\mathcal{R}$ about:

(i) **about the** y**-axis.**

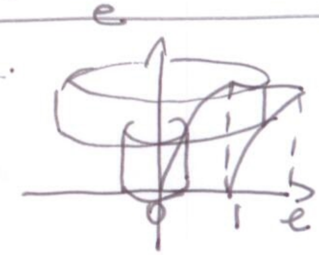


Easier: integrate in y (use washers)

$$\boxed{V = \int_0^1 \pi (e^y)^2 - \pi (y^2)^2\,dy}$$

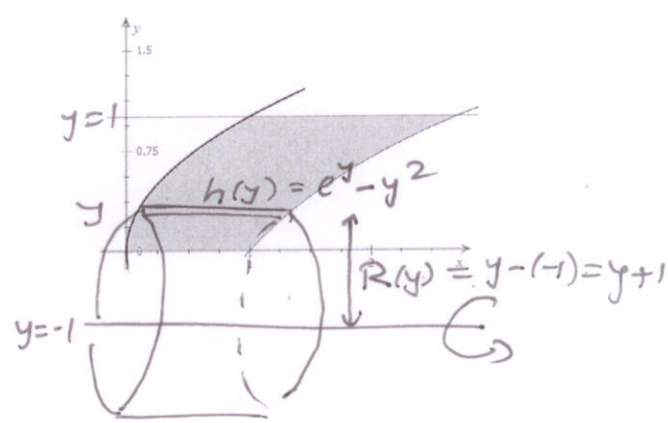
OR

Harder: to integrate in x (shells), we need 2 integrals:



$$\boxed{V = \int_0^1 2\pi x \sqrt{x}\,dx + \int_1^e 2\pi x (1 - \ln x)\,dx}$$

(ii) about the **horizontal line** $y = -1$.



Easier: integrate in y (shells)

$$V = \int_0^1 2\pi \underbrace{R(y)}\,\underbrace{h(y)}\,dy$$
$$= \boxed{\int_0^1 2\pi (y+1)(e^y - y^2)\,dy}$$

OR harder: integrate in x (washers)

$$\boxed{V = \int_0^1 \pi (\sqrt{x}+1)^2 - \pi (1)^2\,dx + \int_1^e \pi (2)^2 - \pi (\ln x + 1)^2\,dx}$$