Name__________________________________

Math 125 **First Midterm** **8:30 Jan. 31, 2019**

(6 problems, 80 minutes, 100 points, 1 sheet of notes but no calculator, no cellphone, no watch permitted)

Please show all your work clearly, and box your final answers. Leave your answers in exact form, but make any obvious simplifications for full credit. Cross out any work that you don't want us to consider. If you run out of space on the problem page, use the back of that page.

1. (15 points) Let

$$f(x) = \frac{1}{4x-3} \qquad \text{and} \qquad g(x) = \frac{1}{x^2}.$$

(a) Find the points of intersection of the two curves. Between these two points, which curve is above the other?

(b) Find the area between the two curves.

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2. (10 points) How far does an object travel in the first 2 minutes if its velocity in km/min is

$$v(t) = \frac{1}{2^t}\ ?$$

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3. (30 points) Evaluate

$$\text{(a)} \int \frac{(\text{Arcsin}(1-3x))^3}{\sqrt{6x-9x^2}}\,dx \qquad \text{(b)} \int_1^{3^{1/6}} \frac{x^2}{1+x^6}\,dx.$$

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4. (15 points) The curve $y = \sqrt{9 + x^2}$ for $-5 \le x \le 5$ is the top of a stage opening; the bottom of the stage is the x-axis. The units are meters. At time $t = 0$ the opening is covered, but at that moment a curtain starts to uncover the stage starting from $x = -5$ and moving to the right. The curtain starts from rest and accelerates at 2 m/sec^2.

(a) Find the equation for the position of the curtain at time t, and then write an integral for the area that's been uncovered at time t.

(b) Find the rate (in square meters per second) at which the uncovered area is increasing when $t = 3$ sec.

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5. (15 points) Let R be the region in the plane that's bounded below by the x-axis, on the left by the y-axis, on the right by the line $x = 2$, and above by a continuous function $y = f(x)$.

(a) Using the method of cylindrical shells, write a definite integral for the volume of the solid obtained by rotating R around the line $x = -3$.

(b) Suppose that we let α_i for $i = 0, 1, 2, 3, 4, 5, 6$ denote the $f(x)$-values as follows:

$$\alpha_0 = f(0), \quad \alpha_1 = f(\frac{1}{3}), \quad \alpha_2 = f(\frac{2}{3}), \quad \alpha_3 = f(1), \quad \alpha_4 = f(\frac{4}{3}), \quad \alpha_5 = f(\frac{5}{3}), \quad \alpha_6 = f(2).$$

Write the 6-th trapezoid rule approximation to the integral in part (a), in terms of the α_i.

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6. (15 points) Write the area of one "hump" of the curve $y = \cos(\frac{\pi}{4}x)$ (contained between $x = -2$ and $x = 2$) as a limit.

Midterm Answers, Jan. 31, 2019

1. (a) Cross-multiplying in $\frac{1}{4x-3} = \frac{1}{x^2}$ and moving everything to the left, we get $x^2 - 4x + 3 = 0$ with roots $x = 1,\ 3$. Since $f(2) = 1/5$ and $g(2) = 1/4$, it follows that $g(x)$ is above.

(b) $\int_1^3 \frac{dx}{x^2} - \int_1^3 \frac{dx}{4x-3} = -x^{-1}\ \Big|_1^3 - \frac{1}{4}\ln(4x-3)\ \Big|_1^3 = \frac{2}{3} - \frac{1}{4}\ln 9 = \frac{2}{3} - \frac{1}{2}\ln 3$. (The last step is optional.)

2. Distance traveled $= \int_0^2 2^{-t}dt = \int_0^2 e^{-(\ln 2)t}dt = -\frac{1}{\ln 2}e^{-(\ln 2)t}\ \Big|_0^2 = -\frac{1}{\ln 2}2^{-t}\ \Big|_0^2 = \frac{3}{4\ln 2}$ km.

3. (a) If we substitute $u = \text{Arcsin}(1-3x)$, then $\frac{du}{dx} = -3(1-(1-3x)^2)^{-1/2} = -3(6x - 9x^2)^{-1/2}$, and so the integral becomes $-\frac{1}{3}\int u^3 du = -\frac{1}{3}\cdot\frac{1}{4}u^4 + C = -\frac{1}{12}(\text{Arcsin}(1-3x))^4 + C$.

(b) If we substitute $u = x^3$, so that $du = 3x^2dx$ and as x goes from 1 to $3^{1/6}$ we have u going from 1 to $\sqrt{3}$, then the integral becomes $\int_1^{\sqrt{3}} \frac{1}{3}\cdot\frac{du}{1+u^2} = \frac{1}{3}\text{Arctan}(u)\ \Big|_1^{\sqrt{3}} = \frac{1}{3}(\frac{\pi}{3} - \frac{\pi}{4}) = \pi/36$.

4. (a) $A = \int_{-5}^{-5+t^2} \sqrt{9+x^2}\, dx$.

(b) $\frac{dA}{dt} = \sqrt{9+(-5+t^2)^2}\,(2t)$. When $t = 3$ this is $\sqrt{9+4^2}\cdot 6 = 5\cdot 6 = 30$ m^2/sec.

5. (a) $2\pi\int_0^2 (x+3)f(x)dx$. (b) $\frac{2\pi}{3}\left(\frac{3}{2}\alpha_0 + \frac{10}{3}\alpha_1 + \frac{11}{3}\alpha_2 + 4\alpha_3 + \frac{13}{3}\alpha_4 + \frac{14}{3}\alpha_5 + \frac{5}{2}\alpha_6\right)$.

6. $\lim_{n\longrightarrow\infty} \frac{4}{n}\sum_{i=1}^{n} \cos\left(\frac{\pi}{4}(-2 + \frac{4i}{n})\right)$.