Math 125
Midterm 1 (January 30, 2020)

NAME: _____ **Section:** _____

- Time: you have **80 minutes**.
- Please show all work and justify your answers. The final answers must be "reasonably" simplified. For example, a rational number must be given in the form $\frac{a}{b}$ for some integers a and b, but it is ok to have expressions like $\ln 3$ or e^4 in your final answer.
- You are allowed to use calculator (Model TI-30X IIS only) and one *handwritten* (with your own handwriting) 8.5 x 11 inch sheet of notes. Writing allowed on both sides.
- Have your *Husky Card* visible on the desk beside you.
- You may use both sides of the paper.
- Make sure you have **9 pages** and **6 problems** before starting the exam.

Academic integrity is expected of all students at all times. Understanding this, I declare I shall not give, use, or receive unauthorized aid.

SIGNATURE: _____

Problem 1: ___ / 20
Problem 2: ___ / 20
Problem 3: ___ / 20
Problem 4: ___ / 20
Problem 5: ___ / 20
Problem 6: ___ / 20

Total: ___ / 120

Problem 1: Evaluate the following integrals:

(a)

$$\int_{-1}^{1} |x^2 - x| \, dx$$

(b)

$$\int \sqrt{\sqrt{t} + 1} \, dt$$

Problem 2: Find the function $y = f(t)$ satisfying

$$y'' = t + \cos(t) \quad , \quad y(0) = 1 \quad , \quad y'(0) = 0\,.$$

Problem 3: Consider the function

$$f(x) = \int_2^{x^2} \sqrt{1 + \ln\left(\frac{t}{2}\right)}\, dt \ .$$

(a) Evaluate $f'(x)$. Remember you need to show all work and justify your answer.

(b) Compute $f(\sqrt{2})$ and $f'(\sqrt{2})$.

Problem 4: Find the area of the region enclosed by the graphs of $f(x) = x^2 + 2$ and $g(x) = 2x + 5$.

Problem 5: Evaluate the following limit:

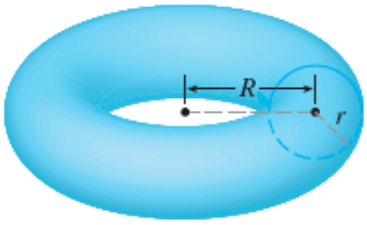
$$\lim_{n \to +\infty} \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{i}{n}}$$

Hint: Use the theory of *Riemann sums* and express the limit as a *definite integral*.

Problem 6: The *torus* (doughnut-shaped solid) in the figure is obtained by rotating the circle $(x - R)^2 + y^2 = r^2$ around the y-axis (assume $R > r$).

(a) Set up an integral for the volume of this torus.

(b) Find the volume of the torus by evaluating the integral.



(extra blank page)

(extra blank page)