Math 125 Autumn 2002 **Quiz #4**

1. (4 points) Find the work required to move a spring from the natural length ($x = 0$) to the position in which the force required to hold it is 10N. Assume that the force is given by $F(x) = kx$ where x is in meters and $F(x)$ is in Newtons.

   Is more work required to do this with a stiff spring ($|k|$ large) or a soft one ($|k|$ small)?

   ANS: For the final position $10 = F(x) = kx$ so $x = \frac{10}{k}$. The work is $W = \int_0^{\frac{10}{k}} kx\,dx = \frac{50}{k}$. More work will be required for a soft spring (you must move it farther).

2. (6 points) Find the following integrals:
   (You may use the fact that $\int \sin^2(x)dx = \frac{-1}{4}\sin(2x) + \frac{x}{2} + C$).

   (a) (2 points) $\int \sin(x)(e^{2x} + 1)dx$

   ANS: $-\cos(x) + \frac{2\sin(x)-\cos(x)}{5}e^{2x} + C$. Using integration by parts twice.

   (b) (2 points) $\int_0^{\pi} \cos^2(x)dx$

   ANS: $\frac{\pi}{2}$.

   (c) (2 points) $\int \frac{x^2}{\sqrt{1-x^2}}dx$

   ANS: Taking $\sin(\theta) = x$ we get $\int \sin^2(\theta)\,d\theta = \frac{-1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2} + C$.