

1. (7 points) Improper integrals:

Determine which of (a) and (b) converges, which diverges and evaluate the convergent one.

(a) (2 points) $\int_1^2 \frac{x}{\sqrt{x-1}} dx$

ANS: Substitute $u = x - 1$ and integrate to get $\lim_{t \rightarrow 0} (\frac{8}{3} - \frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}) = \frac{8}{3}$. Converges.

(b) (2 points) $\int_1^2 \frac{x}{(x-1)^2} dx$

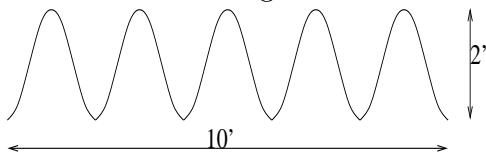
ANS: Substitute $u = x - 1$ and integrate to get $\lim_{t \rightarrow 0} (\frac{1}{t} - \ln(t) - 1)$ which diverges.

Use the comparison theorem to determine whether the following integral converges:

(c) (3 points) $\int_1^\infty \sin^2(x)e^{-x} dx$

ANS: $\sin^2(x) \leq 1$ so $\sin^2(x)e^{-x} \leq e^{-x}$ and $\int_1^\infty e^{-x} dx = \lim_{t \rightarrow \infty} (\frac{1}{e} - \frac{1}{e^t})$ which converges. Thus the original integral converges.

2. (3 points) Write down but **do not evaluate** an integral for the length of tubing required to build a 10 foot long bike rack which x feet from the end has height $1 - \cos(\pi x)$ feet.



ANS: $L = \int_0^{10} \sqrt{1 + \pi^2 \sin^2(\pi x)} dx$.