Your Name (please PRINT clearly)

Student ID #

PLEASE READ these instructions:

- Once the exam starts, check that you have a complete exam: 5 problems on 5 pages of questions.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ page of handwritten notes. Do not share notes.
- Turn OFF your cell phone and put it away in a closed bag or pocket.
- Only a TI-30X IIS calculator is allowed. You may not use headphones or any other electronic devices. Turn around or remove any hats that cover your eyes.
- Unless otherwise instructed, remember to **show your work**. If your work is incorrect, incomplete, or unreadable, you may receive little credit, even if the answer itself happens to be correct.
- Simplify your answers as much as possible but leave them in exact form (e.g. $\pi\sqrt{2}+\frac{1}{2}$), unless otherwise instructed. Place a box around $\boxed{\text{your final answer}}$ to each longer or messier question.
- Please stay within the page borders. Exams will be scanned and the far edges may not be readable.
- **All pages are double-sided, except for this cover page and the last page.** You may use the two blank sides for extra room if needed but if you want us to grade these spare pages clearly **indicate in the problem area** that your work is on the back of the last page or of the cover page.
- Read each question carefully, before and after answering it. Raise your hand if you have a question. Good luck!

Question	Points	Score
1	14	
2	8	
3	10	
4	8	
5	10	
TOTAL	50	

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int e^x\,dx = e^x$
4. $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$
8. $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$
10. $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln|\sec x + \tan x|$
12. $\int \csc x\,dx = \ln|\csc x - \cot x|$
13. $\int \tan x\,dx = \ln|\sec x|$
14. $\int \cot x\,dx = \ln|\sin x|$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
18. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

1. (14 points) Evaluate the following integrals. Show all your steps, and box your final answer.

   (a) $\int \cos^4(x)\,dx$

   (b) $\int \dfrac{e^x}{e^{2x}+8e^x+12}\,dx$

2. (8 points) For each of the four integrals below, just state which of the following methods applies. Your answer should be in one of the following forms:

   - u-substitution, with $u =$... (*specify the substitution to use*)
   - integration by parts, with $u =$..., and $dv =$ ... (*specify the parts to use*)
   - trigonometric substitution, with $x =$... (*specify the trig sub to apply*)
   - partial fractions, with fractions: $\frac{A}{(\ldots)} + \ldots$ (*specify the general decomposition*)

   You do not need to justify or compute anything – and do not evaluate the integrals!

   (a) $\int x^2 \ln(x)\, dx$ Method:

   With:

   (b) $\int x^2 \sec^2(x^3)\, dx$ Method:

   With:

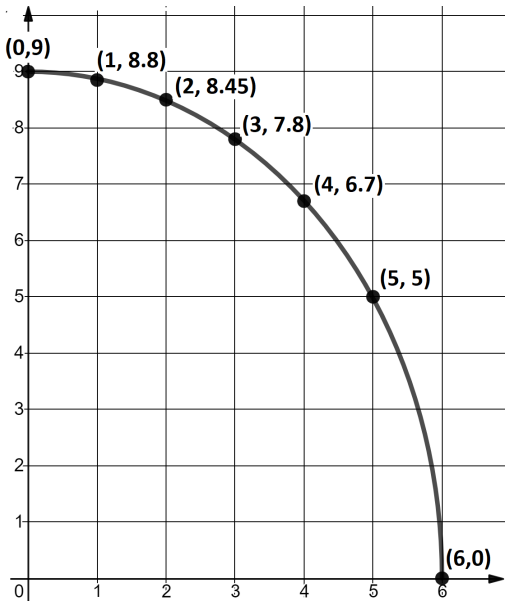
   (c) $\int \frac{-2x+1}{x^4+x^3+x^2}\, dx$ Method:

   With:

   (d) $\int \frac{x^2}{(x^2-4)^{3/2}}\, dx$ Method:

   With:

3. The graph of a function $y = f(x)$ is shown on the right, together with the (x, y)-coordinates of selected points on its graph.



Approximate the **average value** f_{ave} of this function over the interval $[0, 6]$, using the following methods:

(a) (4 points) Midpoint Rule with $n = 3$ subintervals

$M_3 =$

(b) (4 points) Trapezoidal Rule with $n = 3$ subintervals

$T_3 =$

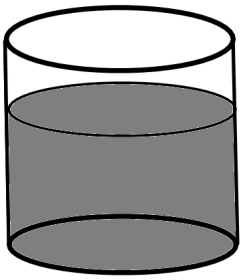
(c) (2 points) How does the actual average value f_{ave} compare to its Trapezoidal and Midpoint approximations? *Circle one of the answers AND one of the justifications below.*

- f_{ave} is higher than both T_n and M_n
- f_{ave} is lower than both T_n and M_n
- $T_n < f_{ave} < M_n$
- cannot tell

Circle a justification for your answer:

- Because the function f is non-negative on $[0, 6]$
- Because the function f is concave-down on $[0, 6]$
- Because the function f decreases on $[0, 6]$
- It depends on n

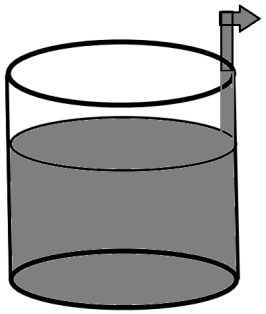
4. (8 points) A cylindrical tank has radius 3 ft and it's 6 feet tall. The tank is partly full with oil, to a height of 4 feet, as shown. The oil in the tank weighs 50 lbs/ft^3.



(a) Consider a **thin horizontal layer** of oil, of thickness Δy, that is **at** y **ft from the bottom of the tank**. Write an expression in y and Δy that is approximately equal to the **work**, in ft-lbs, required to lift just this thin horizontal layer to the top of the tank.

(b) Set up an integral in y equal to the work required to pump all the oil in the tank to the top of the tank. Do not evaluate the integral.

(c) Set up an integral equal to the work required to pump only the top **half** of the oil in the tank to a spout that's 1.5 feet above the top of the tank. Do not evaluate.



5. (10 points) Evaluate the following improper integral. Make sure to use limits and show all your work.

$$\int_0^\infty \frac{1}{(\sqrt{x^2+4x+5})^3}\,dx$$