

**MATH 125C**  
**SECOND MIDTERM EXAM**

Your name \_\_\_\_\_

Your quiz section \_\_\_\_\_

(CA = Folland at 10:30; CB = Hanusa at 10:30; CC = Hanusa at 12:00)

- **Show all your work!**
  - If the answer to a problem is a number such as  $\sqrt{e}$  or  $\ln \pi$ , leave it that way; don't convert it to a decimal.
  - You are assumed to know the indefinite integrals of the following functions:
    - Powers and exponentials  $x^a$  and  $e^x$ .
    - The six basic trig functions:  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\cot x$ ,  $\sec x$ ,  $\csc x$ .
    - Derivatives of the basic trig functions:  $\sec^2 x$ ,  $\csc^2 x$ ,  $\sec x \tan x$ ,  $\csc x \cot x$ .
    - Derivatives of the inverse trig functions:  $1/\sqrt{1-x^2}$  and  $1/(1+x^2)$ .
    - Functions obtained from these by a linear change of variable:  $e^{ax}$ ,  $\sin(x+b)$ , etc.
- All other integrals should be worked out.* You will not receive credit for using other integrals unless you give their derivation.

1. [8 pts] Compute  $\int \frac{x^3}{\sqrt{9-x^2}} dx$ .

2. [8 pts] Compute  $\int \frac{x^3 + 2}{x^2 - 1} dx$ .

3. [8 pts] A 20-foot rope hangs over the edge of a cliff. It rained earlier, so the rope is wet, and since the water tends to seep downwards, the bottom of the rope is wetter (and hence heavier) than the top. Suppose that the weight density of the wet rope at the distance  $y$  feet from the top is  $1 + \frac{3}{80}y$  lb/ft. Calculate the work needed to pull the rope up to the top.

4. [8 pts] Find the average value of  $f(x) = \sin^{-1}(x)$  on the interval  $[\frac{1}{2}, 1]$ .

5. [8 pts] Determine whether the improper integral  $\int_0^{\infty} \frac{24x^2}{(4x^3 + 1)^{4/3}} dx$  converges, and evaluate it if it does.

6. [10 pts] Find the volume of the solid of revolution obtained by revolving the region  $R$  between the curve  $y = e^{2-x}$ , the line  $y = 1$ , and the  $y$ -axis about the  $y$ -axis.