1 (10 points) Evaluate the following indefinite integrals:

(a) (5 points) $\int \frac{x^3}{x^2-4}\,dx$.

ANS: Substitute $u = x^2 - 4$. This gives $\frac{1}{2}x^2 - 2 + 2\ln|x^2-4| + C$.

(b) (5 points) $\int xe^{3x+1}\,dx$.

ANS: Integrate by parts with $u = x$ and hence $dv = e^{3x+1}$. This gives $\frac{1}{9}e^{3x+1}(3x-1)$.

2 (10 points) Evaluate the following definite integrals:

(a) (5 points) $\int_9^{16} \frac{3}{x-2\sqrt{x}}\,dx$.

ANS: Substitute $u^2 = x$. This gives $6\ln(u-2)|_3^4 = 6\ln(2)$.

(b) (5 points) $\int_1^3 \frac{1}{(x^2-4x+5)}\,dx$.

ANS: Complete the square to get $x^2 - 4x + 5 = (x-2)^2 + 1$ and then use inverse trig substitution with the triangle legs being $x-2$ and 1 so that $\tan(\theta) = x - 2$. This gives $\theta|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{2}$.

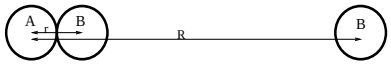
3 (5 points) Determine whether this integral converges or diverges. **Do not evaluate**. $\int_1^\infty \frac{1}{xe^x}\,dx$.

ANS: The function is continuous on the interval $[1,\infty)$ so the improper integral is given by $\lim_{t\to\infty}\int_1^t \frac{1}{xe^x}\,dx$. Use the comparison theorem to show that it converges. If $x \geq 1$ then $0 \leq \frac{1}{xe^x} \leq \frac{1}{e^x}$. Compute that $\lim_{t\to\infty}\int_1^t \frac{1}{e^x}\,dx = \lim_{t\to\infty}(\frac{1}{e} - \frac{1}{e^t}) = \frac{1}{e}$ which converges. Thus by the comparison theorem the given integral also converges.

4 (5 points) Write down but **do not evaluate** an integral for the length of the curve given by the equation $y = \frac{1}{x}$ between the values $x = \frac{1}{2}$ and $x = 2$.

ANS: $y' = -x^{-2}$ so the length is given by $\int_{\frac{1}{2}}^2 \sqrt{1+x^{-4}}\,dx$.

5 (10 points) There is a force between two particles (A and B) given by $F = \frac{1}{1000x^2}$ Newtons where x is the distance in meters between their centers.



(a) (5 points) Write down and evaluate an integral for the work in Joules required to move particle B from a distance r from A to a distance R from A. Your answer should involve the distances r and R.

ANS: $W = \int_r^R \frac{1}{1000x^2}\,dx = \frac{1}{1000}(\frac{1}{r} - \frac{1}{R})$.

(b) (5 points) Find r if the work required to move particle B from a distance r from A to an infinite distance from A is 200 Joules.

ANS: $200 = \lim_{R\to\infty}\frac{1}{1000}(\frac{1}{r} - \frac{1}{R}) = \frac{1}{1000r}$ so $r = \frac{1}{200000}$ meters.

6 (10 points) An odd shaped septic tank is buried ranging from 10 to 22 feet below the ground. You have the cross sectional areas in square feet for each of the depths in the table:

depth in feet	10	12	14	16	18	20	22
area of cross section in square feet	30	20	40	60	50	40	10

(a) (3 points) Use the midpoint rule with 3 intervals to estimate the volume of the tank.

ANS: $M_3 = \frac{22-10}{3}[20 + 60 + 40] = 480$ cubic feet.

(b) (7 points) Use Simpson's rule with 6 intervals to estimate the work which would be needed to pump the contents to the surface if the tank is full and the density of the contents is 70 $\frac{\text{lb}}{\text{ft}^3}$.
Please leave the answer as a long sum.

ANS: $S_6 = 70 \times \frac{22-10}{3\times 6}[10 \times 30 + 4 \times 12 \times 20 + 2 \times 14 \times 40 + 4 \times 16 \times 60 + 2 \times 18 \times 50 + 4 \times 20 \times 40 + 22 \times 10]$ foot pounds.

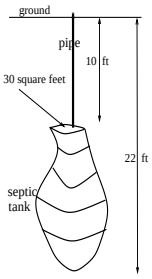


Figure for both parts.