University of Washington **Department of Mathematics**

Midterm Exam #2 **Math 125E** **February 27, 2003**

You are: ____________________ ________ ________________

Name Section Student #

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Instructions

1. Print your **Name, Section** (EA, EB or EC) and **Student #** on this page.
   Do **NOT** separate the pages of the exam.
2. Check to see that your copy of the exam has 6 pages.
3. **SHOW ALL OF YOUR WORK.** Partial credit will only be given where you have made it clear that you understand part of the solution. Answers without justification may not receive full credit. The correct answer may receive no credit if you do not show how you arrived at that answer.
4. You are allowed use of 1 page of handwritten notes. If you need more space to solve a problem, use the back of the page preceding that problem.
5. Read each question carefully. Work the problems in an order that will maximize your score. Be clear and concise. **Good luck!**

1. (10 points) Evaluate the following indefinite integrals. Show all of your work and simplify your answer as much as possible.

(a) (5 points)

$$\int \frac{\cos x}{4 - \sin^2 x}\,dx$$

(b) (5 points)

$$\int \frac{x^3}{\sqrt{4 - x^2}}\,dx$$

2. (10 points) Evaluate the following definite integrals. Show all of your work.

(a) (5 points)

$$\int_0^1 (x^2 + 1)e^{-x}\, dx$$

(b) (5 points)

$$\int_1^4 \frac{e^{1/x}}{x^2}\, dx$$

3. (10 points) The region under the curve

$$y = \cos^2 x \qquad \text{for} \qquad 0 \le x \le \pi/2$$

us rotated about the x-axis. Find the volume of the resulting solid.

4. (10 points) Suppose that at time $t = 10$ seconds an object is traveling at 30.0 m/sec. Its acceleration $a(t)$ is measured at two-second intervals until time $t = 20$, with the following results (the units of acceleration are m/sec^2):

t	10	12	14	16	18	20
$a(t)$	2.3	2.4	2.5	2.6	2.6	2.7

Use the trapezoidal rule with $n = 5$ to estimate the velocity of the object at time $t = 20$.

5. (10 points) The gamma function is defined for all $x > 0$ by

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt.$$

(a) (6 points) Find $\Gamma(1)$ and $\Gamma(2)$
(Note: justify your computation by showing that the corresponding improper integrals converge and evaluating them).

(b) (4 points) Use integration by parts to show that, for positive n

$$\Gamma(n+1) = n\Gamma(n).$$