Math 125C **Midterm 2 Solutions** **Autumn 2003**

1.(a) Let $u = 4y$ and $dv = \sec^2(2y)\,dy$ to get $\int 4y \sec^2(2y)\,dy = 2y\tan(2y) - \int 2\tan(2y)\,dy$
$= 2y\tan(2y) - \ln(\sec 2y) + C$.

(b) $\int \frac{16 + 4x^2 - x^3}{x^3 - 4x^2 + 4x}\,dx = \int -1 + \frac{4}{x} - \frac{4}{x-2} + \frac{12}{(x-2)^2}\,dx$
$= -x + 4\ln|x| - 4\ln|x-2| - \frac{12}{x-2} + C$

2.(a) Let $2t = \sec\theta$ to get $\int_{1/\sqrt{3}}^{1} \frac{dt}{t\sqrt{4t^2-1}} = \int_{\pi/6}^{\pi/3} d\theta = \pi/6$

(b) Let $t^3 = x$ to get $\int_0^{\pi} 3t^2 \sin t\,dt$. Set $u = 3t^2$ and $dv = \sin t\,dt$ to get $-3t^2\cos t\Big|_0^{\pi} + \int_0^{\pi} 6t\cos t\,dt$.
Set $U = 6t$ and $dV = \cos t\,dt$ to get $-3t^2\cos t\Big|_0^{\pi} + 6t\sin t\Big|_0^{\pi} - \int_0^{\pi} 6\sin t\,dt$.
This gives $\left(-3t^2\cos t + 6t\sin t + 6\cos t\right)\Big|_0^{\pi} = 3\pi^2 - 12$

3. Ignore the weight of the rope. Let y be the height of the bag. Then $y = 4t$ feet after t minutes. Let $F(t)$ be the weight of the bag after t minutes. We have $F(t) = 80$ when $y = 20$. This is when $20 = 4t$ or $t = 5$ minutes. Since $F(0) = 160$, the bag is losing $80/5 = 16$ lbs/min. Thus $F(t) = 160 - 16t$ and $F(y) = 160 - 16(y/4) = 160 - 4y$. Therefore the work is given by $\int_0^{20} 160 - 4y\;dy = 2400$ ft-lbs.

4. First let $u = \ln x$ and $dv = x^3\,dx$ to get $\int x^3 \ln x\,dx = \frac{1}{4}x^4\ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx = \frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$.
Now

$$\begin{aligned}
\int_0^1 x^3 \ln x\,dx &= \lim_{a\to 0^+} \int_a^1 x^3 \ln x\,dx \\
&= \lim_{a\to 0^+} \left(\frac{1}{4}x^4\ln x - \frac{1}{16}x^4\right)\Big|_a^1 \\
&= \lim_{a\to 0^+} -\frac{1}{16} - \frac{1}{4}a^4\ln a + \frac{1}{16}a^4 = -\frac{1}{16}
\end{aligned}$$

since $\lim_{a\to 0^+} a^4\ln a = \lim_{a\to 0^+} \frac{\ln a}{a^{-4}} = \lim_{a\to 0^+} \frac{1/a}{-4a^{-5}} = \lim_{a\to 0^+} \frac{-a^4}{4} = 0$ by l'Hôpital's Rule.

5.(a) $dx/dt = \cos t - t\sin t$ and $dy/dt = \sin t + t\cos t$, so the arc length is
$\int_0^{4\pi} \sqrt{(\cos t - t\sin t)^2 + (\sin t + t\cos t)^2}\,dt$. This simplifies to $\int_0^{4\pi} \sqrt{1+t^2}\,dt$.

(b) Let $f(t) = \sqrt{1+t^2}$ and $\Delta x = 4\pi/3$. The Trapezoid Approximation is

$$\frac{2\pi}{3}\Big[f(0) + 2f(4\pi/3) + 2f(8\pi/3) + f(4\pi)\Big] = 81.8766$$