

Problem 1 (6+6 points)

a. Find the general anti-derivative $\int \cos^3 \theta \, d\theta$

b. Evaluate the integral $\int_1^2 \frac{x^3}{x^2 + x + \frac{1}{2}} \, dx$

Problem 2 (6+6 points)

a. Find the general anti-derivative $\int \frac{\sqrt{y-4}}{y} dy$

b. Does the improper integral $\int_0^{\infty} \frac{1}{x^2 + 4x + 3} dx$ converge?
If yes, evaluate it.

Problem 3 (4+4 points).

- a. Find the average value of the function $f(x) = \sqrt{x}$ on the interval $0 \leq x \leq 2$.
- b. Find the number c so that the function $f(x) = \sqrt{x}$ has average value 1 on the interval $[0, c]$.

Problem 4 (8 points).

The following table lists values of two functions, $f_1(x)$ and $f_2(x)$, for different values of x . Use the Trapezoid rule to approximate the area between the curves $f_1(x)$ and $f_2(x)$ for $1 \leq x \leq 4$.

x	1	1.5	2	2.5	3	3.5	4
$f_1(x)$	0	.7	1.1	1.3	1.2	.9	.5
$f_2(x)$	0	-.1	-.2	-.1	0	.1	.3

Problem 5 (10 points).

The line $y = 3x$, for $0 \leq x \leq 1$, is rotated about the y -axis to form a cone (units are in feet). The cone is filled with melted ice cream, which weighs 59.2 lb/ft^3 . How much work does it take to pump all of the ice cream up to the height $y = 10$?