Your Name


Your Signature
$\square$

PLEASE READ the DIRECTIONS below:

- Do not open the test until instructed to do so. This test has 5 problems on 5 pages. Once the test starts, please check that you have a complete exam.
- This exam is closed book. You may use one $8 \frac{1}{2} \times 11$ page of handwritten notes. Do not share notes.
- Only a Ti-30x IIS calculator is allowed. Silence your cell phone and put it away.
- In order to receive credit, you MUST SHOW YOUR WORK. If we cannot tell how you are getting your answers, you may receive little or no credit, even if the answer happens to be correct.
- Simplify your answers as much as possible but leave them in exact form (e.g. $\pi \sqrt{2}+\frac{1}{2}$ ). Do not give decimal approximations, unless otherwise instructed.
- Place a BOX around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Read each question carefully, before and after answering it.

Good luck!

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 12 |  |
| 2 | 12 |  |
| 3 | 8 |  |
| 4 | 9 |  |
| 5 | 9 |  |
| Total | 50 |  |

1. Compute the following definite integrals. Simplify, but leave your answers in exact form.
(a) $[\mathbf{6}$ points $] \int_{\pi / 4}^{\pi / 3} \tan ^{3}(\theta) \sec ^{2}(\theta) d \theta$
(b) $[6$ points $] \int_{1}^{2} x^{3} \ln x d x$
2. Evaluate the following indefinite integrals.
(a) $[\mathbf{6}$ points $] \int \frac{1}{x^{2} \sqrt{x^{2}+25}} d x$
(b) $[\mathbf{6}$ points $] \int \frac{2 x^{3}+1}{x^{3}-x^{2}} d x$
3. 

(a) [3 points] Write an integral expression to compute the average value of $f(x)=\cos \left(x^{2}\right)$ over the interval $[-1,2]$. DO NOT try to compute the integral.
(b) [5 points] Use the Trapezoidal Rule with $n=6$ subintervals to approximate the integral from part (a).
Your answer should either be in exact form (but simplify all you can), or in decimal form with at least 4 digits of precision.
4. [9 points] Consider the improper integral: $\int_{0}^{\infty} \frac{1}{e^{x}+1} d x$.

If it converges, evaluate it. If it diverges, show why. Show all your steps carefully.
5. [9 points] A tank in the shape of the bottom half of a sphere of radius $R=5 \mathrm{~m}$ is partially filled with water, to a height of 3 meters. The density of water is $1000 \mathrm{~kg} / \mathrm{m}^{3}$ and the gravitational constant is $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$. Compute the work necessary to pump all the water to the top of the tank.
(First draw a picture and clearly label your origin and axes. Make sure to show your steps clearly. Include units in your final answer.)

