

Math 125 Autumn 2002**Quiz #7 Solutions**

A large door has a shape bounded on the left by the x-axis, below by the y-axis and above and on the right by the curve $y = 4 - x^2$. The units are meters. The area of the door is $\frac{16}{3}$ square meters.

1. (7 points) Find the centroid of the door.

$$\text{ANS: } \bar{x} = \frac{3}{16}M_y = \frac{3}{16} \int_0^2 x(4 - x^2)dx = \frac{3}{4}. \quad \bar{y} = \frac{3}{16}M_x = \frac{3}{16} \int_0^2 \frac{1}{2}(4 - x^2)^2 dx = \frac{8}{5}.$$

2. (3 points) The door is submerged with the bottom in 5 meters of water and the top in 1 meter. Find the hydrostatic force on the door if the density of water is 1000 kilograms per cubic meter.

$$\text{ANS: } F = AP = A\rho g d = \frac{16}{3}1000(9.8)(5 - \frac{8}{5}) = \frac{533120}{3} \text{ Newtons. With an integral this is } F = \int_0^4 1000(9.8)(5 - y)\sqrt{(4 - y)}dy$$

3. **Extra Credit:** V_x is the volume obtained by rotating the door about the x-axis and V_y is the volume obtained by rotating the door about the y-axis. Use problem (1) to find $\frac{V_x}{V_y}$.

$$\text{ANS: } \frac{V_x}{V_y} = \frac{M_x}{M_y} = \frac{32}{15}.$$