Math 126 **Final Examination** **Spring 2006**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Problem	Total Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Let $f(x) = e^{2x-3}$.

(a) [**5 points**] Find the third-degree Taylor polynomial $T_3(x)$ for $f(x)$ based at $b = 0$.

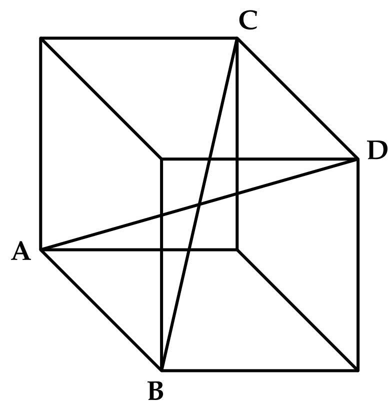
(b) [**5 points**] Give an upper bound on the difference between $T_3(x)$ and $f(x)$ on the interval $(-0.5, 0.5)$.

2.

(a) [**6 points**] Find Taylor series for the function $f(x) = \ln(2 + 5x)$ centered at $b = 0$.

(b) [**4 points**] Give an interval on which your series converges and justify your answer.

3. [**10 points**] Consider a cube such as the one shown in the figure. Consider the line segment connecting corners A and D, and the line segment connecting corners B and C. These are *diagonals* of the cube.



Find the acute angle between these two diagonals.

4. [**10 points**] Find an equation of the plane that contains the following two intersecting lines:

$$x = 4t, \quad y = -1 + 2t, \quad z = 5 - 4t$$

and

$$x - 1 = 1 - y = \frac{z}{3}.$$

5. [**10 points**] Find the point or points on the curve

$$y = x^4$$

at which the curvature is maximum.

6. Consider the curve given by the vector function

$$r(\vec{t}) = \langle \sin(\pi t)\,,\, \cos(\pi t)\,,\, t^2 - t \,\rangle.$$

(a) [**5 points**] Find an equation for the normal plane to the curve when $t = 2$. (The normal plane is perpendicular to the tangent vector.)

(b) [**5 points**] The curve intersects the xy-plane at two points. Write the formula (but do not evaluate it) for the length of the curve between these two points.

7. A particle is moving so that its position at time $t > 0$ is given by the vector

$$\vec{r}(t) = \left\langle t^2 + 2, \frac{1}{t}, t \right\rangle.$$

(a) [**5 points**] Find all times when the particle's velocity and acceleration vectors are orthogonal.

(b) [**5 points**] Find the magnitude (absolute value) of the tangential and normal components of the particle's acceleration vector at the times you found in part (a).

8. [**10 points**] The x-coordinate of a particle in the (x, y) plane is calculated based on the particle's polar coordinates. If the radius r is 2 cm with a possible error of 0.05 cm, and the angle θ is $\frac{\pi}{6}$ with a possible error of 0.1, use differentials to approximate the range of possible values of the x-coordinate.

9. [**10 points**] Find nonnegative numbers x, y and z that minimize the quantity

$$M = x^2 + y^2 + z^2$$

subject to the condition

$$xy^2z^4 = 1.$$

10.

(a) [**5 points**] Write (but do not evaluate) an iterated integral representing the volume contained inside the cylinder $x^2 + y^2 = 4$, above the plane $z = 0$, and below the plane $x - z = 0$.

(b) [**5 points**] Evaluate the iterated integral

$$\int_0^{\frac{\pi}{2}} \int_y^{\frac{\pi}{2}} \frac{\sin x}{x} \, dx \, dy.$$