

MATH 126 – Spring 2007
Final Exam Hints, Answers, and Partial Solutions

1. (a) ANSWER: $T_2(x) = 2 + 4(x - 1) + 3(x - 1)^2$
- (b) HINT: Taylor's inequality states that the error is bounded by $\frac{M}{3!}|x - 1|^3$, where M is an upper bound of $f'''(x)$ on the interval J . But $f'''(x) = 6$ for all values of x . So, we can take M to be 6 and we need an interval J on which $|x - 1|^3 < 0.001$.
- ANSWER: $J = [0.9, 1.1]$

2. HINT: The Taylor series for $\frac{e^{x^2} - 1}{x}$ is

$$x + \frac{x^3}{2!} + \frac{x^5}{3!} + \frac{x^7}{4!} + \dots$$

Further, since $\frac{3}{(x-1)^2} = 3 \frac{d}{dx} \left(\frac{1}{1-x} \right)$, the Taylor series for $\frac{3}{(x-1)^2}$ is

$$3 + 6x + 9x^2 + 12x^3 \dots$$

ANSWER: The Taylor series for $f(x)$ is

$$3 + 7x + 9x^2 + 12.5x^3 + \dots$$

3. (a) F; (b) T; (c) F; (d) T; (e) F; (f) T; (g) T; (h) F

4. (a) ANSWER: $x = -1, y = t, z = 3 + t$

(b) ANSWER: $5x - 2y + 2z = 1$

5. ANSWER: 1

6. (a) ANSWER: $t = \sqrt[6]{\frac{1}{2}}$

(b) ANSWER: $\frac{-\frac{2}{t^5} + 4t}{\sqrt{1 + \frac{1}{t^4} + 4t^2}}$

7. (a) ANSWER: $y = \frac{3}{7}(x - 2) + 2$

(b) ANSWER: The curve has a horizontal tangent at $t = -\frac{1}{2}$ and vertical tangents at $t = 0$ and $t = -\frac{3}{4}$.

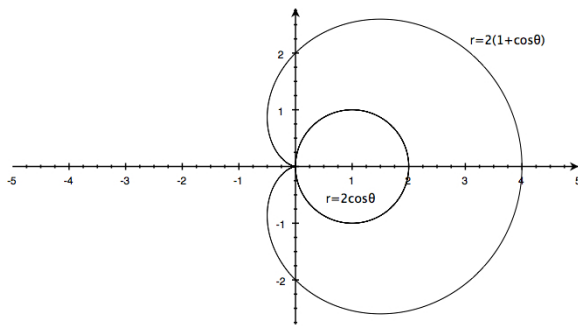
8. (a) ANSWER: The domain of f is the set of points (x, y) such that $y < 2x$ but $y \neq 2x - 1$.

(b) ANSWER: $z = -2e(x - e) + 5e(y - e) + 3e^2$

(c) ANSWER: $f(3, 3) \approx 9e$

9. ANSWER: The base of the glass side is 8.0505 meters and the height of the glass side is 4.0252 meters. The length of the pool is 30.85989 meters.

10. (a) ANSWER: The region R is the region between these two curves:



(b) ANSWER: 5π