Math 126 **Final Examination** **Winter 2006**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides may be used).
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	

Problem	Total Points	Score
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. [**10 points**] Find an approximation for the value of

$$I = \int_1^2 e^{1-x^2} dx.$$

by replacing the integrand e^{1-x^2} above with its quadratic (or second Taylor polynomial) approximation based at $b = 1$, and then integrating the result.

2. [**10 points**] Expand

$$f(x) = \ln\left(\frac{1+x}{1-x}\right)$$

in a Taylor series about $x = 0$. You must express your answer using summation notation.

3. [**10 points**] Find a vector $\vec{a}$ such that $\vec{a}$ is orthogonal to $< 1, 5, 2 >$ and has length equal to 6.

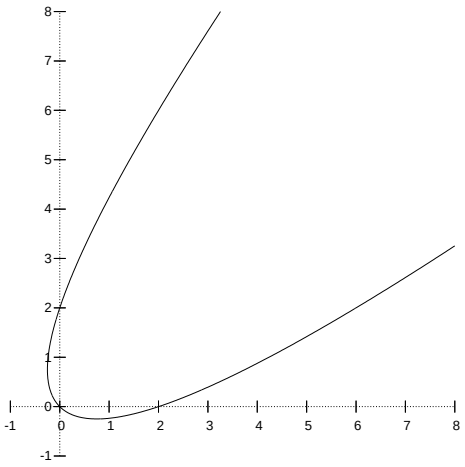
4. [**10 points**] Find the line that is the intersection of the two planes

$$3x - y + z = 6$$

and

$$x + y - 5z = 1.$$

5. A particle is moving so that its position at time t is given by $x = t^2 + t, \quad y = t^2 - t$.



(a) [**4 points**] Find the line which is tangent to the path of the particle at time $t = -2$.

(b) [**6 points**] Find the time(s) when the tangent line will pass through the point $(0, 3)$.

6. Consider the surface defined by $z = f(x, y)$ where

$$f(x, y) = xy + x + y^2.$$

(a) [**5 points**] Find the tangent plane to this surface at the point $(1, 2, 7)$.

(b) [**5 points**] Using a linear approximation, give an approximate value of $f(1.1, 2.1)$.

7. [**10 points**] Find the value of d so that the plane

$$2x + y + z = d$$

is a tangent plane to the surface

$$z = f(x, y) = x^2 + y^2.$$

8. [**10 points**] Evaluate

$$I = \int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dxdy.$$

9. [**10 points**] The base of a pile of sand covers the region in the xy-plane that is bounded by the parabola $x^2 + y = 6$, the line $y = x$, and the positive x-axis. The depth of the sand above the point (x, y) in this region is x^2. Find the volume of the sand pile in the form of an integral. **Do not evaluate the integral**.

10. [**10 points**] The position of a particle is given by

$$\mathbf{r}(t) = (t^4 + 2t^2 + 1)\,\mathbf{i} + (1 + 4t - t^4)\,\mathbf{j}.$$

Find the cosine of the angle between the position and acceleration vectors of the particle when $t = 1$.