

MATH 126 – Winter 2007
Final Exam Hints, Answers, and Partial Solutions

1. (a) ANSWER: $T_1(x) = \frac{1}{5} + \frac{1}{25}x$
(b) HINT: Taylor's inequality states that the error is bounded by $\frac{M}{2}x^2$, where M is an upper bound for $|f''(x)|$. Here, $|f''(x)| = \frac{2}{|5-x|^3}$, which is largest on I when $x = 2$. So, we can take M to be $\frac{2}{27}$.
ANSWER: error $\leq \frac{4}{27}$
(c) HINT: Show that $f^{(n+1)}(x) = \frac{n!}{(5-x)^{n+1}}$ and that the error is bounded by $\frac{1}{3} \left(\frac{2}{3}\right)^{n+1}$.
Then solve the system $0.04 < \frac{1}{3} \left(\frac{2}{3}\right)^{n+1} < 0.05$ for n .
ANSWER: $n = 4$
2. (a) HINT: Substitute $-2x^2$ into the Taylor series for $\frac{1}{1-x}$ and $3x$ into the Taylor series for $\cos x$ and subtract.
ANSWER: $\sum_{k=0}^{\infty} (-1)^k \left(2^k - \frac{3^{2k}}{(2k)!}\right) x^{2k}$
(b) HINT: The Taylor series for $\cos x$ converges for all x , but the Taylor series for $\frac{1}{1-x}$ converges only for x such that $|x| < 1$. This means that the Taylor series for $f(x)$ converges only for x such that $|2x^2| < 1$.
ANSWER: $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$
(c) ANSWER: $T_4(x) = \frac{5}{2}x^2 + \frac{5}{8}x^4$
3. (b) HINT: $\iint_R x + \sqrt{x^2 + y^2} dA = \int_0^{\pi/2} \int_4^5 (r \cos \theta + r) r dr d\theta$
ANSWER: $\frac{61}{3} \left(1 + \frac{\pi}{2}\right)$
4. ANSWER: $x = -t$, $y = 3 - 7t$, $z = 4 + 2t$
5. ANSWER: $(0, 0)$ gives a saddle point; $(1, 1)$ gives a local max
6. ANSWER: $f(1.1, 1.2) \approx 0.3$
7. ANSWER: $y = x - \sqrt{2}$
8. (a) HINT: Note that $\vec{T}(t)$ and $\vec{r}'(t)$ have the same direction. So showing that $\vec{r}(t) \cdot \vec{r}'(t) = 0$ is sufficient.
(b) HINT: The vector $\vec{r}'(0)$ is a normal vector to the normal plane at the point $(1, 0, 0)$.
ANSWER: $x + y + z = 1$
9. (a) ANSWER: $\vec{a}(t) = \langle 0, 5, 4 \rangle$ and $\vec{v}(t) = \langle 1, 5t - 3, 4t - 2 \rangle$.
(b) ANSWER: $\vec{r}(1) = \langle 1, -\frac{1}{2}, 0 \rangle$
(c) ANSWER: $a_T = \frac{28}{3}$, $a_N = \sqrt{5}$
(d) ANSWER: $\kappa(1) = \frac{\sqrt{5}}{9}$