Math 126 **Final Examination** **Spring 2005**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes.
- Give your answers in exact form. Do not give decimal approximations.
- Graphing calculators are not allowed. Do not share notes.
- In order to receive credit, you must show your work. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	6	
3	10	
4	8	
5	12	
6	6	

Problem	Total Points	Score
7	6	
8	10	
9	14	
10	8	
11	8	
Total	100	

1. [**12 points total**] Consider the function $f(x) = \sin\left(\frac{\pi x}{6}\right)$.

(a) [**6 points**] Find $T_2(x)$, the second order Taylor polynomial for $f(x)$ centered at $a = 1$.

(b) [**6 points**] Use Taylor's Inequality to find an upper bound on $|f(1.1) - T_2(1.1)|$.

2. [**6 points total**] Determine whether or not the infinite series $\sum_{n=1}^{\infty}(-1)^n\, n\, e^{-n}$ converges. Justify your answer.

3. [**10 points total**] Let L_1 be the line given by the parametric equations

$$x = 2t,\ y = 0,\ z = 4 - 4t,$$

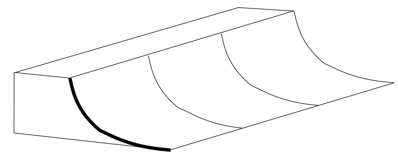
and let L_2 be the line given by the parametric equations

$$x = 2 - 2u,\ y = 3u,\ z = 0.$$

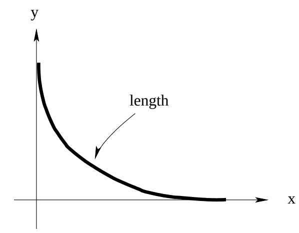
(a) [**4 points**] Find the point of intersection of L_1 and L_2.

(b) [**6 points**] Find an equation of the plane that contains both L_1 and L_2. Give your answer in the form $ax + by + cz = d$.

4. [**8 points**] The curved portion of a skateboard ramp has a cross section given by the inverted cycloid (for maximal speed at the bottom):



$$
\begin{aligned}
x &= r\,(t - \sin(t)) \\
y &= r\,(1 + \cos(t))
\end{aligned}
$$



for $0 \le t \le \pi$. Find r so that the length of the curved portion is 8 feet. (Hint: To compute the integral, you might find the trig identity $1 - \cos\theta = 2\sin^2(\theta/2)$ useful.)

5. [**12 points total**] You are assigned the task of building the electronics for an antenna on a spacecraft so that the antenna will always point toward a space station. On board are accelerometers which measure the acceleration in the x, y and z direction. Choose a coordinate system so that the space station is at the origin $(0, 0, 0)$, and assume that the spacecraft starts at the space station with initial velocity $\langle 0, 0, 0\rangle$. The acceleration at time t is given by

$$\mathbf{a}(t) = \left\langle e^t,\ \cos(t),\ t \right\rangle .$$

(a) [**4 points**] Find the velocity as a vector function of time.

(b) [**4 points**] Find the position as a vector function of time.

(c) [**4 points**] Find a unit vector (which depends on time) that points from the spacecraft to the space station at time t.DO NOT SIMPLIFY YOUR ANSWER.

6. [**6 points total**] Using the fact that $\dfrac{d}{du}(\tan^{-1} u) = \dfrac{1}{1+u^2}$, find the Maclaurin series for $g(x) = \tan^{-1}\left[(x/2)^3\right]$.

7. [**6 points total**] Find a vector $\mathbf{u}$ which satisfies both of the following conditions:

(i) $\mathbf{u}$ is orthogonal to $\langle 2, 1, 4 \rangle$,

(ii) the cross product of $\mathbf{u}$ and $\langle 1, 2, 0 \rangle$ equals $\langle 2, -1, 0 \rangle$

8. [**10 points total**] Let C be the curve defined by the vector function

$$\mathbf{r}(t) = \left\langle e^t \sin t,\ e^t \cos t,\ e^{-2t} \right\rangle.$$

(a) [**4 points**] Find a vector equation of the tangent line at $t = 0$. (Be sure your answer is an equation.)

(b) [**6 points**] Find the curvature κ at $t = 0$.

9. [**14 points total**] Let $f(x, y) = \ln\left(x^2 + \sqrt{y}\right) + e^y \cos x$.

(a) [**3 points**] Find the partial derivative f_x.

(b) [**3 points**] Find the partial derivative f_y.

(c) [**4 points**] Find the second order partial derivative f_{xy}.

(d) [**4 points**] Find an equation of the tangent plane to the graph of $z = f(x, y)$ at the point $(0, 1, e)$.

10. [**8 points total**] Evaluate the iterated integral $\displaystyle\int_0^1 \int_{x^2}^1 x \sin(\pi y^2)\, dy\, dx.$

11. [**8 points total**] Evaluate the double integral $\iint_D x^2 + x + y^2 \, dA,$ where D is the region

$$D = \left\{ (x, y) \ : \ x^2 + y^2 \leq 4 \text{ and } y \geq x \right\}.$$