Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around $\boxed{\text{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	15	
3	12	
4	8	
5	9	

Problem	Total Points	Score
6	10	
7	14	
8	12	
9	10	
Total	100	

1. (10 points) Indicate whether each of the following is True or False. You do not need to explain your answer.

   (a) ______ Suppose the linear approximation of a function $z = f(x, y)$ at the point $(2, -3, 5)$ is $L(x, y) = 5$. Then $(2, -3)$ is a critical point of $f(x, y)$.

   (b) ______ The set of points $\{(x, y, z) : x^2 + y^2 = 4\}$ is a circle.

   (c) ______ The dot product $\mathbf{proj_u v} \cdot \mathbf{u}$ is zero for any two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$.

   (d) ______ The line $\mathbf{r}(t) = \langle 3-4t, 5-6t, -2+t\rangle$ is parallel to the plane $-4x-6y+z-10 = 0$.

   (e) ______ The cross product of any two nonzero vectors is always nonzero.

   (f) ______ There is a function $g(x, y)$ such that $g_x(x, y) = x + 4y$ and $g_y(x, y) = 3x - y$.

   (g) ______ If D is the domain given by $x^2 + y^2 \leq 1$, then $\int\int_D \sqrt{1 - x^2 - y^2}dA < \pi$.

   (h) ______ Let $f(x, y) = x^3 - 3xy - y^3 + 10$. Then f has a saddle point at $(0, 0)$.

   (i) ______ If $z = f(x, y)$ has a minimum value at (a, b), then $f_x(a, b) = f_y(a, b) = 0$.

   (j) ______ Let $\mathbf{r}(t)$ be a vector function. Then the unit normal vector $\mathbf{N}(t)$ is equal to $\mathbf{T}(t) \times \mathbf{B}(t)$.

2. (15 points) Let P_1 be the plane $x + y + z = 3$ and P_2 be the plane $x - 2y + 4z = 0$.

   (a) Find the angle between these two planes.

   (b) Find the distance from the point $(1, 2, 3)$ to the plane P_1.

THIS QUESTION CONTINUES ON THE NEXT PAGE.

(c) Again, P_1 is the plane $x + y + z = 3$ and P_2 is the plane $x - 2y + 4z = 0$. Find a linear equation for the plane that contains the point $(1, 2, 3)$ and the line of intersection of P_1 and P_2.

3. (12 points) A particle is moving along a curve C in 3-dimensional space with position vector

$$\mathbf{r}(t) = \langle t, f(t), g(t)\rangle,\ \ t \geq 0,$$

for some differentiable functions $f(t)$ and $g(t)$.

It is indicated by some experimental data that

- the initial position of the particle is at the origin $\langle 0, 0, 0\rangle$;
- the velocity vector of the particle is orthogonal to the vector $\langle 2, 1, 0\rangle$ for any $t \geq 0$; and
- the particle is moving on the surface $z = 2x^2 + y^2$ (this surface is an elliptic paraboloid). That is, the entire curve C is on this surface.

(a) Find the functions $f(t)$ and $g(t)$.

(b) Find the tangential and normal components of the acceleration for any $t \geq 0$.

(c) Find the curvature $\kappa(t)$ of $\mathbf{r}(t)$ at any $t \geq 0$.

4. (8 points) Consider

$$f(x,t) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/2t}, \qquad \text{for all } t > 0,\ -\infty < x < \infty..$$

Show that

$$f_t(x,t) = \frac{1}{2} f_{xx}(x,t).$$

5. (9 points) The dimensions of a closed rectangular box are measured as 80 cm, 60 cm, and 20 cm respectively, with a possible error of 0.2 cm in each dimension. Use differentials to estimate the maximum error in calculating the surface area of the box.

6. (10 points) An aquarium is an open-topped, rectangular box. The base of the aquarium is made of slate while the sides are made of glass. Slate costs five times as much (per unit area) as glass. Find the dimensions of the aquarium with fixed volume V that minimize the cost of the material. (For full credit, you must show some work or write a few sentences that demonstrate that the dimensions you find give the *minimum* cost.)

7. (14 points)

   (a) Sketch the region of integration and change the order of integration:

$$\int_1^e \int_{\ln x}^{-x+(1+e)} f(x,y)\,dy\,dx.$$

   (b) A lamina occupies the region $D = \{(x,y) : x^2 + y^2 \leq 25, y \geq x\}$ and has density $\rho(x,y) = e^{-\sqrt{x^2+y^2}}$. Find the mass of the lamina.

8. (12 points) Let $f(x) = 4x^2 - 5x + \ln(3x)$.

   (a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 1$.

   (b) Let a be a real number such that $0 < a < 1$ and let J be the closed interval $[1 - a, 1 + a]$. Use the Quadratic Approximation Error Bound to find an upper bound for the error $|f(x) - T_2(x)|$ on the interval J.

   (c) Find a value of a such that $|f(x) - T_2(x)| \leq 0.01$ for all x in $J = [1 - a, 1 + a]$.

9. (10 points) Let $g(x) = \dfrac{1}{5x-2} + \dfrac{1}{3-7x}$.

   (a) Find the Taylor series for $g(x)$ based at 0. Write the series using one Σ sign.

   (b) Find the interval on which the series in (a) converges.