Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 9 problems. on 10 pages.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	8	
3	12	
4	8	
5	10	

Problem	Total Points	Score
6	14	
7	14	
8	10	
9	12	
Total	100	

1. (12 points) Consider the triangle determined by the three points $P(1, 2, 3)$, $Q(2, 3, 0)$, and $R(3, 5, 1)$.

   (a) Show that these three points form a right triangle.

   (b) Give an equation for the plane containing these three points. Write your answer in the form $ax + by + cz = d$.

   (c) Give a parametric representation for the intersection of the plane you found in part (b) and the cylinder $x^2 + y^2 = 4$.

2. (8 points) Consider the two planes $3x - 2y + 3z = 5$ and $4y - 3z = -4$.

   (a) Give a parametric representation of the line of intersection of these two planes.

   (b) Give the equation of the plane that contains the origin and is perpendicular to the line in part (a).

3. (12 points) Answer the following multiple choice questions. You need not show any work.

(a) Consider the line $\mathbf{r}(t) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$ passing through the point $P_0(x_0, y_0, z_0)$ and let $P_1(x_1, y_1, z_1)$ be a point not on this line. If $Q(\bar{x}, \bar{y}, \bar{z})$ is the closest point on the line to P_1, then

i. $\langle \bar{x}, \bar{y}, \bar{z} \rangle = \langle x_0, y_0, z_0 \rangle + \text{Proj}_{\langle a,b,c \rangle}\langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

ii. $\langle x_0 - \bar{x}, y_0 - \bar{y}, z_0 - \bar{z} \rangle \cdot \langle x_1 - \bar{x}, y_1 - \bar{y}, z_1 - \bar{z} \rangle = 0$

iii. Both (i) and (ii) are true.

iv. Neither (i) and (ii) is true.

ANSWER: ____________

(b) Let $P_0(x_0, y_0, z_0)$ be a point in $\mathbf{R}^3$ and $\mathbf{n} = \langle a, b, c \rangle$ be a vector in $\mathbf{R}^3$. Consider the plane $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$. Then the closest point in the plane to the point $P_1(x_1, y_1, z_1)$ must lie on the line $\mathbf{r}(t) = \langle x_1 - ta, y_1 - tb, z_1 - tc \rangle$.

i. True

ii. False

ANSWER: ____________

(c) Let $\mathbf{v}$ and $\mathbf{w}$ be two non-zero vectors in $\mathbf{R}^3$ such that $|\mathbf{v} \times \mathbf{w}| = 0$. Which of the following statements must be true? (Give all correct responses.)

i. $|\mathbf{v} \cdot \mathbf{w}| = 0$

ii. $|\text{Proj}_{\mathbf{v}}\mathbf{w}| = |\mathbf{w}|$

iii. $|\text{Proj}_{\mathbf{v}}\mathbf{w}| = |\mathbf{v}|$

iv. $|\mathbf{v} \cdot \mathbf{w}| = |\mathbf{v}|\,|\mathbf{w}|$

ANSWER: (list all) ____________

4. (8 points) Consider the cone $z = \sqrt{x^2 + y^2} + 1$. Suppose a particle travels along the trajectory $\mathbf{r}(t) = \langle 1 - t, 1 - t, 1 \rangle$, where $t \geq 0$ represents time. Find the point on the cone that is closest to the particle's position at time t.

   (Your answer should be a point (x, y, z) whose coordinates depend on t.)

5. (10 points) Find the moment about the y-axis of a lamina in the shape of the region in the xy-plane enclosed by the curves $y = 0$, $y = x^2$, and $x = 1$ if its density at the point (x, y) is $\rho(x, y) = e^y$.

6. (14 points) Consider the parametric curve $\mathbf{r}(t) = \langle 2\cos t, -3\sin t, t\rangle$.

   (a) For which values of t does the curve intersect the surface $9x^2 + 4y^2 = 36z^2$?

   (b) Are there any values of t for which the binormal vector to the curve at time t is parallel to the z-axis? If so, find them. If not, explain why not.

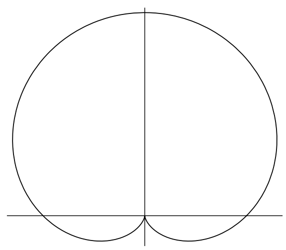
   (c) Compute the tangential component of the acceleration for this motion as a function of t.

   (d) For how many values of t is the tangential component of the acceleration equal to 0?

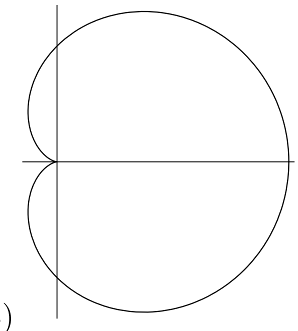
7. (14 points) Consider the polar curve given by $r = 1 + \sin(\theta)$.

   (a) Which is the following curves is the graph of $r = 1 + \sin(\theta)$? You need not show any work. (Circle the letter corresponding to your answer.)

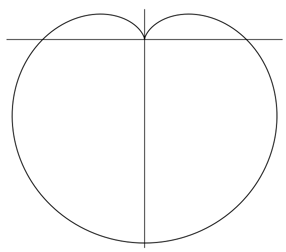
(A)



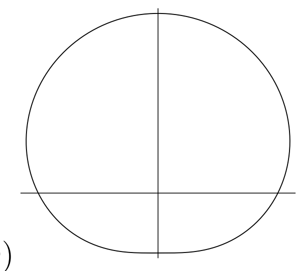
(B)



(C)



(D)



   (b) Find the area between the x-axis and the portion of the curve $r = 1 + \sin(\theta)$ that lies below the x-axis.

THIS PROBLEM CONTINUES ON THE NEXT PAGE.

(c) Find all horizontal tangent lines to the curve $r = 1 + \sin(\theta)$.

8. (10 points) Find the first four non-zero terms of the Taylor series based at $b = 0$ for

$$f(x) = e^{x^3} - \frac{1}{1+x^2}.$$

9. (12 points) Let $f(x) = e^x \sin x$.

   (a) Find $T_2(x)$, the second Taylor polynomial for $f(x)$ based at $b = 0$.

   (b) Find an upper bound for $|f(x) - T_2(x)|$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and use the methods of this course to justify your answer.