Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 8 pages.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	11	
5	12	

Problem	Total Points	Score
6	14	
7	15	
8	12	
Total	100	

1. (12 points) Below are a series of multiple choice questions. Circle the correct answer. You do not need to show your work.

   (a) Let $\mathcal{P}$ be the plane given by the equation $2x + y - z = 5$. Choose the sentence that best describes the plane $\mathcal{Q}$ given by the equation $2x + y - z = 10$.

   a. $\mathcal{Q}$ is parallel to $\mathcal{P}$. **b.** $\mathcal{Q}$ is equal to $\mathcal{P}$. **c.** $\mathcal{Q}$ intersects $\mathcal{P}$ in a line.

   (b) Consider the set of points C whose distance to the origin is twice the distance to the x-axis. Which of the following best describes C?

   a. a cone **b.** an elliptic paraboloid **c.** a hyperboloid of two sheets **d.** none of these

   (c) Suppose $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| > 0$. Then $\mathbf{a}$ and $\mathbf{b}$ are

   a. parallel. **b.** perpendicular. **c.** both the zero vector.

   (d) Which of the following surfaces does the curve $\mathbf{r}(t) = \langle t, t, 2t^2 \rangle$ lie on?

   a. $x^2 + y^2 = z^2$ **b.** $x = 0$ **c.** $x^2 + y^2 = z$ **d.** all of these **e.** none of these

2. (12 points) Consider the vector function $\mathbf{r}(t) = \langle t, \cos(t), \sin(t) \rangle$.

   (a) Compute the velocity vector $\mathbf{v}(t)$ and the unit tangent vector $\mathbf{T}(t)$.

   (b) Compute the unit normal vector $\mathbf{N}(t)$.

   (c) Compute the curvature of $\mathbf{r}(t)$ at the point $t = 1209.17$.

3. (12 points) Find and classify all critical points of

$$f(x,y) = 2x^3 + 9xy^2 + 15x^2 + 27y^2.$$

4. (11 points) Suppose you need to know an equation of the plane tangent to a surface S at the point $P(-7, 4, -33)$. You don't have an equation for S but you know that the following curves lie on S:

$$\mathbf{r}_1(t) = \langle 2t - 9, 5 - t, -3t^2 + 26t - 56 \rangle \text{ and } \mathbf{r}_2(u) = \langle -\sqrt{u}, \sqrt{u - 33}, -33 \rangle.$$

Find the equation of the plane tangent to S at P.

Simplify your final answer to the form $z = Ax + By + C$.

5. (12 points) Evaluate the definite integral

$$\int_0^1 \int_{e^y}^{e} \frac{\sqrt{1 + (\ln x)^2}}{x} dx\ dy.$$

6. (14 points) Find the volume of the solid that is above the xy-plane, within the sphere $x^2 + y^2 + z^2 = 1$, and below the cone $z = \sqrt{x^2 + y^2}$.

7. (15 points) Let $f(x) = \sqrt{x^3}$.

   (a) Find the second Taylor polynomial $T_2(x)$ based at $b = 1$.

   (b) Find an upper bound for $|T_2(x) - f(x)|$ on the interval $[1 - a, 1 + a]$. Assume $0 < a < 1$. Your answer should be in terms of a.

   (c) Find a value of a such that $0 < a < 1$ and $|T_2(x) - f(x)| \leq 0.004$ for all x in $[1 - a, 1 + a]$.

8. (12 points) The Basic Taylor Series are:

$$\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \qquad \cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \qquad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k.$$

(a) Use the list of Basic Taylor Series to find the Taylor series for $f(x) = x^4 \arctan(x^3)$ based at $b = 0$. (Use Σ-notation.)

(b) Find the open interval on which the series in part (a) converges.

(c) Find $f^{(2017)}(0)$. (That is, the 2017^{th} derivative of f at 0.) Give an exact answer.