Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

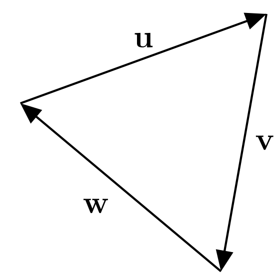
- CHECK that your exam contains 9 problems on 8 pages.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	8	
2	6	
3	12	
4	12	
5	12	

Problem	Total Points	Score
6	13	
7	12	
8	13	
9	12	
Total	100	

1. (8 points) Let **u**, **v**, and **w** be three-dimensional head-to-tail **unit** vectors in the xy-plane, forming an equilateral triangle, as pictured. Compute the following:

   (There is enough information provided to compute a specific value or vector as an answer to each of these questions.)



   (a) $\mathbf{u} \cdot \mathbf{u} =$

   (b) $\mathbf{u} \cdot \mathbf{v} =$

   (c) $\mathbf{u} + \mathbf{v} + \mathbf{w} =$

   (d) $|\mathbf{u} \times (\mathbf{v} \times \mathbf{w})| =$

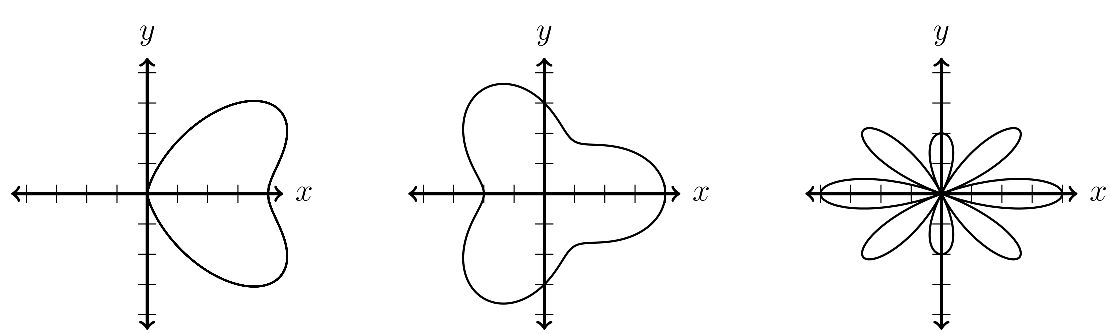
2. (6 points) Answer the following (unrelated) questions. No need to justify your answer.

   (a) Give an example of the equation of a plane that is perpendicular to the plane $z = x - y$. Write your answer in the form $Ax + By + Cz = D$.

   (b) Which of the following is a level curve (i.e. a trace when $z = k$, a constant) for the surface $x^2 - y^2 + z = 0$? Circle all that apply.

   ELLIPSE / PARABOLA / HYPERBOLA / PAIR OF LINES.

   (c) One of these is the graph of $r = 5\cos(\theta) - \cos(5\theta)$. Circle it.

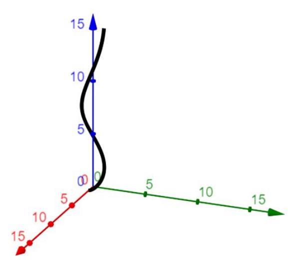


3. (12 points) Consider the helix-like curve $\mathbf{r}(t) = \left\langle \cos(t), \sin(t), \frac{2t^{3/2}}{3} \right\rangle,\ t \geq 0.$

   (a) Write a Cartesian equation for a surface that contains this curve.

   (b) Compute the arc length of this curve from $t = 0$ to $t = a$, where a is an arbitrary positive constant.

   (c) Compute the curvature $\kappa(t)$ of this curve, and determine $\lim_{t\to\infty} \kappa(t)$.



4. (12 points) Find and classify all critical points of the function $f(x, y) = 4x^3 - 2x + 2xy - y^2$.

5. (12 points) Let $z = f(x, y)$ be the function defined by the implicit equation

$$z\sqrt{x^2 + y^2} + e^{z+1} + y = 0.$$

(a) Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at the point $(3, 4, -1)$.

(b) Use linear approximation to estimate the value of $f(3.07, 4.12)$.

6. (13 points) Suppose $f(x, y)$ is continuous and D is a region in the xy-plane such that

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{y-2}^{-\sqrt{y}} f(x,y)\,dx\,dy.$$

(a) Sketch D and reverse the order of integration.

(b) Let D be the region described above and suppose a lamina in the shape of D has variable density $\rho(x, y) = -12x$. Compute the mass of the lamina.

(You do not need to use the same setup from part (a) to compute this.)

7. (12 points) Use polar coordinates to find the volume of the solid below the cone $\sqrt{x^2+y^2} = 5z$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 10y$.

8. (13 pts) For **ALL** parts below, let $f(x) = \dfrac{4x}{2x+1} - xe^{6x}$ and $b = 0$.

   (a) Give the Taylor series for $f(x)$ based at b.
   Give your final answer in Sigma notation using one Sigma sign.

   (b) Give the open interval of convergence for your answer in part (a).

   (c) Use the first three nonzero terms of the Taylor series to estimate the value of $f\left(\frac{1}{10}\right)$.
   Give your final answer to three digits after the decimal.

9. (12 pts) For **ALL** parts below, consider Taylor polynomials for $g(x) = e^{x/2}$ based at $b = 1$.

   (a) Find the third Taylor polynomial, $T_3(x)$, for $g(x)$ based at $b = 1$.

   (b) On the interval $I = [0, 2]$, for which of the values of n below does Taylor's inequality guarantee that $|f(x) - T_n(x)| < 0.001$?
   You **must** show enough error bound calculations to justify your answer.

   Circle **ALL** that apply: $n = 2$ $n = 3$ $n = 4$ $n = 5$ n = 6