Your Name


Student ID \#


Professor's Name


Your Signature
$\square$

TA's Name


- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8 \frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Give your answers in exact form. Do not give decimal approximations.
- In order to receive credit, you must show your work.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 10 |  |
| 3 | 12 |  |
| 4 | 10 |  |
| 5 | 12 |  |


| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 6 | 10 |  |
| 7 | 12 |  |
| 8 | 12 |  |
| 9 | 12 |  |
| Total | 100 |  |

1. (10 points) Find parametric equations of the line of intersection of the planes

$$
-x+y+4 z=7 \text { and } x-3 y+2 z=15 .
$$

2. (10 points) Let $a$ be a positive real number.
(a) Find the curvature of the vector function

$$
\vec{r}(t)=\langle 2 \cos t, a \sin t, t\rangle
$$

at $t=\pi$. Simplify as much as possible.
(b) Find the positive value of $a$ for which $\vec{r}(t)$ has curvature $\frac{1}{41}$ at $t=\pi$.
3. (12 points)
(a) Find the equation of the tangent plane to the surface $z=x^{2} y+e^{y}-\frac{y}{x}$ at the point $(1,0,1)$.
(b) Find all points $\left(x_{0}, y_{0}, z_{0}\right)$ on the surface $z=x^{3} y^{2}$ at which the tangent plane is parallel to the plane $3 x+18 y-z=0$.
4. (10 points) Suppose $x, y$, and $z$ are positive numbers. What is the minimum possible value of

$$
S=x+2 y+3 z
$$

if $x y z=100 ?$ (Include in your solution some verification that your answer is the minimum.)
5. (12 points)
(a) Let $R$ be the region in the first quadrant bounded by $y=5-4 x^{2}$ and $y=x^{2}$. Evaluate the integral

$$
\iint_{R} x y d A
$$

(b) Evaluate the integral:

$$
\int_{0}^{3} \int_{y^{2}}^{9} y e^{x^{2}} d x d y
$$

6. (10 points) Find the area of the region in the first quadrant inside the cardioid $r=1+\cos \theta$ and outside the circle $r=2 \cos \theta$.

7. (12 points) Let $f(x)=e^{2 x-4}$.
(a) Give the third Taylor polynomial $T_{3}(x)$ for $f(x)$ based at $b=2$.
(b) Use Taylor's Inequality to bound the error $\left|f(x)-T_{3}(x)\right|$ on the interval $I=[1,3]$.
8. (12 points)
(a) Find the Taylor series for $\cos \left(2 x^{3}\right)$ based at $b=0$. Write your answer in sigma notation.
(b) Use the first three nonzero terms of the Taylor series in part (a), to approximate the value of the integral $\int_{0}^{1} \cos \left(2 x^{3}\right) d x$. (Give your final answer to four digits after the decimal point).
9. (12 points)
(a) Find the Taylor series for $f(x)=\frac{1}{1+4 x}-\frac{1}{6 x-3}$ based at $b=0$. Write your final answer using one sigma sign.
(b) Give the first three non-zero terms of the Taylor series from part (a).
(c) Give an interval $I$ where the Taylor series from part (a) converges. (Show your work)
