Your Name


Student ID \#


Professor's Name


Your Signature
$\square$


TA's Name


- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8 \frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 12 |  |
| 3 | 10 |  |
| 4 | 12 |  |
| 5 | 10 |  |


| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 6 | 10 |  |
| 7 | 12 |  |
| 8 | 12 |  |
| 9 | 12 |  |
| Total | 100 |  |

1. (10 points) Consider the vector function $\mathbf{r}(t)=\left\langle 6 t, 8 t, 5 t^{2}\right\rangle$. Find the coordinates of the points where the graph has curvature equal to $\frac{1}{80}$.
2. (12 points) Let $\ell$ be the line of intersection of the planes $x+y=0$ and $x-y+z=1$. Find the equation of the plane that contains $\ell$ and is perpendicular to the line

$$
\frac{x-1}{7}=2-y=\frac{z-3}{4} .
$$

3. (10 points) Find the point of intersection of the lines tangent to the curve given by

$$
\mathbf{r}(t)=\langle\sin \pi t, 2 \sin \pi t, 5 \cos \pi t\rangle
$$

at the points where $t=0$ and $t=\frac{1}{2}$.
4. (12 points) Let $z=f(x, y)=\frac{\sqrt{4-x^{2}}}{\sqrt{y^{2}-9}}$.
(a) Find and sketch the domain of $f(x, y)$ on the axes below.

(b) Use the total differential to approximate the change in $z$ if $x$ changes from 0 to 0.1 and $y$ changes from 5 to 5.2 .
5. (10 points) Evaluate the double integral $\int_{0}^{\sqrt{2}} \int_{y^{2}}^{2} y^{3} e^{x^{3}} d x d y$
6. (10 points) Let $R$ be the region in the $x y$-plane which lies in the second quadrant (that is, $x \leq 0$ and $y \geq 0$ ) and is inside the circle $x^{2}+y^{2}=4$. Find the volume of the solid above $R$ and below the surface $z=x^{2}+y^{2}+y$.
7. (12 points) You are standing on a surface where the height is given by

$$
f(x, y)=50+x y-3 x-\frac{1}{4} y^{4} .
$$

Assume the postive $x$-axis points east and the positive $y$-axis points north.
(a) Find and classify all critical points of $f$.
(b) You are standing at a particular point $\left(x_{0}, y_{0}, z_{0}\right)$ on the surface. At this point, if you face due EAST the slope is -1 and if you face due NORTH the slope is 2 . Find the equation for the tangent plane to the surface at the point.
8. (12 points) Consider the function $f(x)=x^{2} \sin (x)$.
(a) Find the Taylor series for $f(x)$ based at $b=0$. Write your answer using sigma notation.
(b) Find the Taylor series based at $b=0$ for

$$
F(x)=\int_{0}^{x} f(t) d t
$$

Write your answer using sigma notation.
(c) Find the sixth Taylor polynomial of $F(x)$ based at $b=0$.
9. (12 points) Consider the function $f(x)=\ln \left(\frac{x+2}{5}\right)+x^{3}$.
(a) Find the second Taylor polynomial $T_{2}(x)$ for $f(x)$ based at $b=3$.
(b) Find an upper bound on the error $\left|T_{2}(x)-f(x)\right|$ on the interval $[1,5]$.
(c) What is the smallest value of $\left|T_{2}(x)-f(x)\right|$ on the interval $[1,5]$ ?

