Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 9 problems.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of hand-written notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	12	
3	10	
4	10	
5	12	

Problem	Total Points	Score
6	10	
7	10	
8	12	
9	14	
Total	100	

1. (10 points)

   (a) Find the equation for the plane containing the points $(0,0,1)$, $(1,2,3)$, $(2,4,5)$, and $(4,8,11)$.

   (b) Consider the tangent plane to the surface $z = x^2 + xy - 3y$ at the point $(2,3,1)$. Compute the angle of intersection between this plane and the plane from part (a).

2. (12 points) Consider the surface S defined by the equation $xy = z$.

   (a) True or False: the traces of S parallel to the xy-plane are lines.

   (b) True or False: the traces of S parallel to the yz-plane are lines.

   (c) True or False: the traces of S parallel to the xz-plane are lines.

   (d) Find all points (x, y, z) at which the curve $\langle x(t), y(t), z(t)\rangle = \langle \sin(t), \cos(t), \sin(2t)\rangle$ intersects S.

3. (10 points) Two particles travel along the space curves

$$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle, \quad \mathbf{r}_2(t) = \langle 1 + 4t, 1 + 16t, 1 + 52t \rangle.$$

   (a) Find the points (x, y, z) at which their paths intersect (if any such points exist).

   (b) Do these particles ever collide? Explain.

4. (10 points) The osculating plane at every point on the curve

$$\mathbf{r}(t) = \left\langle t+2, 1-t, \frac{t^2}{2} \right\rangle$$

is the same plane. Find an equation for it.

5. (12 points) Find the absolute maximum and minimum values of the function

$$f(x, y) = xy^2 - y^2 - x$$

on the region D where

$$D = \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 9\}.$$

6. (10 points) Evaluate the following double integral

$$\int_0^1 \int_0^{\arctan y} \frac{\sin x}{1+y^2}\, dx\, dy.$$

7. (10 points) A lamina occupies the region D in the first quadrant bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 3$, and the x-axis and the y-axis. Find the y-coordinate of the center of mass of the lamina if the density at any point in D is given by

$$\rho(x, y) = \frac{x}{x^2 + y^2 + 1}.$$

8. (12 points) Let $f(x) = e^{x^2}$.

   (a) Find the second Taylor polynomial $T_2(x)$ based at $b = 1$.

   (b) Find an upper bound for $|T_2(x) - f(x)|$ on the interval $[0, 2]$.

   (c) What is the smallest value of $|T_2(x) - f(x)|$ on the interval $[0, 2]$?

9. (14 points) Let $F(x) = \displaystyle\int_0^x \frac{t^3}{9+t^2}\,dt.$

(a) Write the Taylor series for the function $F(x)$ based at $b = 0$. (Use Σ-notation.)

(b) Find the open interval on which the series in part (a) converges.

(c) Find $F^{(10)}(0)$. Give an exact answer.