Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 8 pages.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	13	
3	15	
4	14	
5	12	

Problem	Total Points	Score
6	10	
7	12	
8	14	
Total	100	

1. (10 points) Each of the following multiple choice problems has one correct answer. Circle it. You do not need to show any reasoning.

   (a) Suppose $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{proj_a b} = \mathbf{a} \times \mathbf{b}$. Then...

   (i) $\mathbf{b} = \mathbf{a}$. (ii) $\mathbf{b} = -\mathbf{a}$. (iii) $\mathbf{b} = \mathbf{0}$. (iv) $\mathbf{b}$ is orthogonal to $\mathbf{a}$, but $\mathbf{b} \neq \mathbf{0}$.

   (b) Let $\mathcal{P}$ be the plane through $(0, 0, 0)$, $(1, 1, 2)$, and $(2, 0, 0)$. Then $\mathcal{P}$ also contains...

   (i) $(1, 2, 4)$. (ii) $(1, 4, 2)$. (iii) $(2, 1, 4)$. (iv) $(2, 4, 1)$.

   (c) Let ℓ be the line $\mathbf{r}(t) = \langle 4t, 2t, 2 + t\rangle$ and let $\mathcal{P}$ be the plane $x - 3y + 2z = 4$. Then ℓ is...

   (i) parallel to $\mathcal{P}$. (ii) orthogonal to $\mathcal{P}$. (iii) contained in $\mathcal{P}$. (iv) none of these.

   (d) Suppose $\mathcal{S}$ is the set of points P such that the distance from P to the z-axis is 1. Then $\mathcal{S}$ is...

   (i) two planes. (ii) a cone. (iii) a circular paraboloid. (iv) a cylinder.

   (e) Let $\mathbf{T}(t)$ be the unit tangent vector for $\mathbf{r}(t)$, and suppose $|\mathbf{r}'(0)| = \frac{1}{2}$. Then...

   (i) $\mathbf{T}(0) = 2\mathbf{r}'(0)$. (ii) $\mathbf{T}(0) = \frac{1}{2}\mathbf{r}'(0)$. (iii) $\mathbf{T}(0) = \mathbf{r}'(0)$. (iv) $\mathbf{T}(0) = \mathbf{0}$.

2. (13 points) Let $\mathbf{r}(t) = \langle t\sin(\pi t), t\cos(\pi t), 4 - t\rangle$ and let $\mathcal{S}$ be the surface $x^2 + y^2 = z^2$.

   (a) Find the point where $\mathbf{r}(t)$ intersects the surface $\mathcal{S}$.

   (b) Write parametric equations for the tangent line to $\mathbf{r}(t)$ at $t = 1$.

   (c) Find the curvature of $\mathbf{r}(t)$ at $t = 0$.

3. (15 points)

   (a) Find the second partial f_{xy} for $f(x, y) = \ln(y) + ye^{x^2y}$.

   (b) Find $\dfrac{\partial z}{\partial x}$ for $z^4 - z = x\sin(y) + \dfrac{1}{y} + x^y$.

   (c) Use the linear approximation to $g(x, y) = \sqrt{10 - x^2 - 2y^2}$ at $(2, 1)$ to approximate the value of $g(2.01, 0.95)$.

4. (14 points) Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $3x + 2y + z = 12$.

   In order to receive full credit, you should show some work and/or write a couple of sentences explaining why your answer is indeed the maximum.

5. (12 points) Let R be the region in the xy-plane bounded by the curve $y = x^{1/3}$ and the line $y = x$ in the first quadrant. Compute the double integral

$$\iint_R \cos(y^2)\, dA.$$

6. (10 points) Find the volume of the solid that is under the surface

$$z = e^{x^2+y^2}$$

and above the region

$$R = \{(x, y) \mid x \geq 0, x^2 + y^2 \leq 4\}.$$

7. (12 points) Let $f(x) = x^{\frac{1}{3}}$.

   (a) Find the second Taylor polynomial for $f(x)$ based at $b = 8$.

   (b) Use Taylor's inequality to find an upper bound for the error $|f(x) - T_2(x)|$ on the interval $[7, 9]$.

   (c) Use you answer in part (a) to estimate the cube root of 9. Give your answer as a fraction or a decimal rounded to three digits after the decimal.

8. (14 points) Write down the Taylor series for the following functions.

   For each,

   - write your answer in sigma notation;
   - write out the first three nonzero terms explicitly; and
   - give the interval on which it converges.

   (a) $f(x) = \dfrac{2}{4x^2 + 9}$ based at $b = 0$.

   (b) $f(x) = \dfrac{\cos(2x^2) - 1}{5x^4}$ based at $b = 0$.