Your Name


Student ID \#

Professor's Name


Your Signature
$\square$


TA's Name


- CHECK that your exam contains 8 problems on 8 pages.
- This exam is closed book. You may use one $8 \frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 12 |  |
| 3 | 12 |  |
| 4 | 14 |  |
| 5 | 14 |  |


| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 6 | 12 |  |
| 7 | 12 |  |
| 8 | 14 |  |
| Total | 100 |  |

1. (10 points) Each of the following multiple choice problems has one correct answer. Circle it. You do not need to show any reasoning.
(a) Suppose $\operatorname{comp}_{\mathbf{a}} \mathbf{b}=\frac{1}{2}|\mathbf{b}|$. Then the angle between $\mathbf{a}$ and $\mathbf{b}$ is...
(i) $30^{\circ}$.
(ii) $45^{\circ}$.
(iii) $60^{\circ}$.
(iv) $90^{\circ}$.
(b) Suppose $\mathcal{S}$ is the set of points $P$ such that the distance from $P$ to the $x$-axis is equal to 3. Then $\mathcal{S}$ is...
(i) a plane.
(ii) a cylinder.
(iii) a sphere.
(iv) a cone.
(c) The surface $z=x^{2}+2 x y$ is tangent to the plane $z=6 x+4 y-8$ at the point...
(i) $(-2,3,-8)$
(ii) $(0,2,0)$.
(iii) $(2,1,8)$.
(iv) $(4,0,16)$.
(d) The value of $\int_{2}^{5} \int_{3}^{5}\left(5+\sin ^{2}\left(y x^{2}+y^{3}\right)\right) d y d x$ is between...
(i) 0 and 10 .
(ii) 10 and 20 .
(iii) 20 and 30 .
(iv) 30 and 40 .
(e) The Taylor series for $f(x)=\frac{1}{2-x^{2}}$ centered at $b=0$ converges on the interval...
(i) $(-1,1)$.
(ii) $(-2,2)$.
(iii) $(-4,4)$.
(iv) $(-\sqrt{2}, \sqrt{2})$.
2. (12 pts) Let $L$ be the line of intersection of the two planes

$$
x+y+2 z=c \quad \text { and } \quad x-c y-c z=-1
$$

where $c$ is some real number. Find a value of $c$ for which $L$ is perpendicular to the plane $3 x-y-z=0$.
3. (12 pts) Find the curvature of the ellipse

$$
x=3 \cos (t), \quad y=4 \sin (t), \quad z=1,
$$

at the points $(3,0,1)$ and $(0,4,1)$.
4. (14 pts) Find and classify all the critical points of $f(x, y)=4 x y-3 y+\frac{1}{x}-\frac{1}{4} \ln (y)$.

Clearly show your work in using the second derivative test and label your answers.
5. (14 pts) Compute the volume of the solid between the surface $x^{2}+y+z=4$ and the $x y$-plane above the first quadrant.
6. (12 pts) Compute

$$
\iint_{R} e^{-\left(x^{2}+y^{2}\right)} d A
$$

where $R=\left\{(x, y): x^{2}+y^{2} \leq 9\right\}$.
7. $(12 \mathrm{pts})$ Let $f(x)=1+x+x^{2}+3 x^{3}$.
(a) Find the second-degree Taylor polynomial, $T_{2}(x)$, for $f(x)$ based at $b=1$.
(b) Determine an interval around $b=1$ on which

$$
\left|T_{2}(x)-f(x)\right|<0.024
$$

8. (14 pts) Let $f(x)=\frac{x^{3}}{1+x^{4}}$.
(a) Find the Taylor series for $f(x)$ based at zero. Express your answer using sigma notation.
(b) Use the Taylor series you found in (a) to find the Taylor series for

$$
g(x)=x^{2} \ln \left(1+x^{4}\right) .
$$

Express your answer using sigma notation.

