Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 9 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, scientific calculator. Do not share notes or calculators.
- Give your answers in exact form. Do not give decimal approximations.
- In order to receive credit, you must show your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	12	
3	10	
4	12	
5	14	

Problem	Total Points	Score
6	8	
7	12	
8	12	
9	10	
Total	100	

1. (10 points) Find the angle of intersection of the curves given by

$$\mathbf{r}_1(t) = \left\langle t+4, t^2-2, 3t+5 \right\rangle$$

and

$$\mathbf{r}_2(s) = \left\langle s+3, s^2+s-7, 2s^2 \right\rangle.$$

2. (12 points) Consider the curve traced by the vector function

$$\mathbf{r}(t) = \langle 5\cos t, 12\cos t, 13\sin t\rangle .$$

(a) Compute $\mathbf{T}(t)$.

(b) Compute $\mathbf{N}(t)$.

(c) Compute $\mathbf{B}(t)$.

(d) Compute the curvature as a function of t.

3. (10 points) Decide if the following are TRUE or FALSE. You do not need to explain your answer.

(a) ______ The vectors $< 1, 2, -1 >$ and $< 2, -1, 3 >$ are orthogonal.

(b) ______ If two lines in space are not parallel, then they intersect at some point in space.

(c) ______ The set of points $\{(x, y, z) : x^2 + y^2 = 1\}$ is a circle.

(d) ______ In a contour graph of $z = f(x, y)$, the contour lines corresponding to different values do not intersect.

(e) ______ The cross product $\mathbf{proj_u v} \times \mathbf{u} = 0$ for any two vectors $\mathbf{u}$ and $\mathbf{v}$.

(f) ______ The set of all points equidistant from $(1, 2, 3)$ and $(2, 3, 6)$ is a plane.

(g) ______ If $f(x, y)$ is a continuous function such that $f(x, y) \leq 9$ for all (x, y) in a closed bounded domain D, then $\int\int_D f(x, y)dA \leq 9\text{Area}(D)$.

(h) ______ If $f_x(1, 2) = 0$ and $f_y(1, 2) = 0$, then $f(1, 2)$ must be a local minimum or a local maximum.

(i) ______ Let $f(x, y)$ be a continuous function. Then

$$\int_0^1 \int_x^{\sqrt{x}} f(x, y)dydx = \int_0^1 \int_y^{y^2} f(x, y)dxdy.$$

(j) ______ The Taylor series of a function $f(x)$ is convergent at all x in the domain of $f(x)$.

4. (12 points) Let l_1 be the line of intersection of the planes $x + 2y - z = 2$ and $3x + 6y - z = 14$. Let l_2 be the line perpendicular to the plane $2x - y = 7$ at the point $(3, -1, 5)$.

   (a) Find parametric equations for the line l_1.

   (b) Find parametric equations for the line l_2 and verify that l_1 and l_2 are parallel.

   (c) Find the equation of the plane containing l_1 and l_2.

5. (14 points) You are standing at the point $x = y = 100$ feet on a hillside whose height above sea level is given by

$$f(x, y) = 1000 + \frac{1}{1000}(3x^2 - 5xy + y^2)$$

with the positive x axis pointing East and the positive y axis pointing North.

(a) If you head due East, will you initially be ascending or descending? At what angle from the horizontal?

(b) If you head due North, will you initially be ascending or descending? At what angle from the horizontal?

(c) Find the equation of the plane that contains the soles of your shoes. i.e. the tangent plane to where you are standing.

(d) Use your answer in part (c) to *approximate* your altitude after you have walked 2 feet North from your original position.

6. (8 points) Find the x-, y-, and z-coordinates of all points on the hyperboloid

$$z^2 = 1 + 2x^2 + y^2$$

that are closest to $Q = (0, 1, 0)$.

7. (12 points) Consider the function $f(x) = x^2 e^{x-1}$.

   (a) Find the second Taylor polynomial T_2 of $f(x)$ based at $b = 1$.

   (b) Use the second Taylor polynomial T_2 to approximate $f(0.9)$.

   (c) Use Taylor's inequality to find an upper bound for the error in your approximation in part (b).

8. (12 points) Consider the function

$$f(x) = \int_0^x \frac{\sin t}{t} dt.$$

(a) Find the Taylor series for the function $f(x)$ about $b = 0$. You must express your answer using summation notation.

(b) Find the first three nonzero terms in part (a).

(c) Find the interval on which the Taylor series of $f(x)$ converges. Justify your answer.

9. (10 points) Find the average value of the function

$$f(x, y) = e^{-(x^2+y^2)}$$

over the region

$$D = \{(x, y)|1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0\}$$

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