• This exam contains 10 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.

• This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, non-programmable scientific calculator. Do not share notes or calculators.

• Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)

• In order to receive full credit, you must show all of your work.

• Place a box around YOUR FINAL ANSWER to each question.

• If you need more room, use the backs of the pages and indicate to the reader that you have done so.

• Raise your hand if you have a question.
1. (8 points) Indicate whether each of the following is true (T) or false (F). Circle your answer. No justification for your answer is needed.

(a)  T  F  Two planes that are not parallel must intersect in space.

(b)  T  F  Two lines that are not parallel must intersect in space.

(c)  T  F  For any two vectors $\mathbf{v}$ and $\mathbf{w}$, we always have $\mathbf{v} \times \mathbf{w} = \mathbf{w} \times \mathbf{v}$.

(d)  T  F  If the curvature of a curve in space is constant, then the curve must be a circle.

(e)  T  F  For any function $f(x, y)$,
$$\int_0^3 \int_{3x}^3 f(x, y) \, dy \, dx = \int_0^3 \int_{3y}^3 f(x, y) \, dx \, dy.$$

(f)  T  F  For all real values of $x$, the function $\frac{1}{1-x}$ is equal to its Taylor series based at $b = 0$.

(g)  T  F  Every function $f(x)$ has a Taylor series based at $b = 0$.

(h)  T  F  If $f(x, y)$ is a nonzero continuous function over a region $R$, then
$$\iint_R |f(x, y) - 1| \, dA \leq \iint_R (|f(x, y)| - 1) \, dA.$$
2. (10 points) In the parallelogram below, we know the points $A(2, 1, 2)$ and $B(4, 2, 3)$ and the vector $\overrightarrow{AD} = (3, 0, 4)$.

(a) Compute the area of the parallelogram.

(b) Find the coordinates of the intersection point $E$ of the two diagonals.
3. (10 points) Given the point $P(2, 4, 0)$ and the line $l$ given by parametric equations $x = 1 - 2t,$ $y = 2 - t,$ $z = 3t,$ answer the following.

(a) Find the equation of the plane that contains the point $P$ and the line $l.$

(b) Find the parametric equations of the line through the point $P$ which is perpendicular to the line $l.$
4. (12 points) Consider the vector function

\[ r(t) = (1 + 13 \sin t, 12 \cos t, 1 - 5 \cos t) \, . \]

(a) Find the speed and the normal and tangential components of acceleration at time \( t \) of the particle whose position is given by \( r(t) \).

(b) Compute the curvature at time \( t \).

(c) Find the equation of the line tangent to the curve at \( t = \pi \).
5. (10 points) Consider the function \( f(x, y) = \sin(x) \cos(y) \).

(a) Let \( R = \{(x, y) : 0 \leq x \leq \pi \text{ and } 0 \leq y \leq \pi \} \). Find all critical points of \( f \) that lie in the region \( R \) and classify them according to type (max, min, saddle, other).

(b) Let \( D \) be the triangular region bounded by the lines \( x = \frac{\pi}{2}, \ y = \frac{\pi}{2}, \) and \( y = \frac{\pi}{2} - x \). Find the locations of the absolute max and min of \( f \) on \( D \).

(You may find the identity \( \cos\left(\frac{\pi}{2} - x\right) = \sin(x) \) useful.)
6. (10 points) Let $f(x, y) = x^y + y^x$.

(a) Calculate the partial derivatives $f_x$ and $f_y$.

(b) Find the tangent plane to the surface $z = f(x, y)$ in $\mathbb{R}^3$ at the point $(1, 1, 2)$.

(c) Use linear approximation to estimate the value of $f(1.01, 0.99)$. 
7. (6 points) Consider the surface $S$ in $\mathbb{R}^3$ given by

$$x^3 + y^3 + z^3 = 0.$$

(a) Find the tangent plane to $S$ at the point $(1, 0, -1)$.

(b) Describe the shape of the intersection of $S$ and the plane $z = 0$. 
8. (10 points) Find the center of mass of the lamina that occupies the region

$$D = \{(x, y) : 4 \leq x^2 + y^2 \leq 16 \text{ and } y \geq |x|\}$$

with density function $\rho(x, y) = y + e^{\sqrt{x^2+y^2}}$. 
9. (12 points) Consider the function \( f(x) = \ln(x^2 + 3x) \).

(a) Find the Taylor series for \( f(x) \) based at \( b = 1 \). Write your answer using sigma notation.

(b) Find the Taylor series based at \( b = 1 \) for

\[
F(x) = \int_1^x f(t) \, dt.
\]

Write your answer using sigma notation.

(c) Find the 5th Taylor polynomial of \( F(x) \) based at \( b = 1 \).
10. (12 points) Consider the function $f(x) = 2x - x^2 + e^{2x^2 - x}$.

(a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 0$.

(b) Find an upper bound on the error $|T_2(x) - f(x)|$ on the interval $[-1, 1]$.

(c) What is the smallest value of $|T_2(x) - f(x)|$ on the interval $[-1, 1]$.