Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- This exam contains 10 problems. CHECK THAT YOU HAVE A COMPLETE EXAM.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a non-graphing, non-programmable scientific calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	8	
2	10	
3	10	
4	12	
5	10	

Problem	Total Points	Score
6	10	
7	6	
8	10	
9	12	
10	12	
Total	100	

1. (8 points) Indicate whether each of the following is true (T) or false (F). Circle your answer. No justification for your answer is needed.

(a) **T** **F** Two planes that are not parallel must intersect in space.

(b) **T** **F** Two lines that are not parallel must intersect in space.

(c) **T** **F** For any two vectors $\mathbf{v}$ and $\mathbf{w}$, we always have $\mathbf{v} \times \mathbf{w} = \mathbf{w} \times \mathbf{v}$.

(d) **T** **F** If the curvature of a curve in space is constant, then the curve must be a circle.

(e) **T** **F** For any function $f(x, y)$,

$$\int_0^3 \int_{3x}^3 f(x,y)\,dy\,dx = \int_0^3 \int_{3y}^3 f(x,y)\,dx\,dy.$$

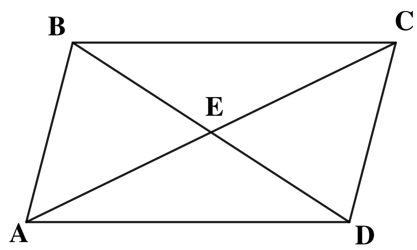
(f) **T** **F** For all real values of x, the function $\dfrac{1}{1-x}$ is equal to its Taylor series based at $b = 0$.

(g) **T** **F** Every function $f(x)$ has a Taylor series based at $b = 0$.

(h) **T** **F** If $f(x, y)$ is a nonzero continuous function over a region R, then

$$\iint_R |f(x,y) - 1|\,dA \leq \iint_R (|f(x,y)| - 1)\,dA.$$

2. (10 points) In the parallelogram below, we know the points $A(2, 1, 2)$ and $B(4, 2, 3)$ and the vector $\overrightarrow{AD} = \langle 3, 0, 4 \rangle$.



(a) Compute the area of the parallelogram.

(b) Find the coordinates of the intersection point E of the two diagonals.

3. (10 points) Given the point $P(2,4,0)$ and the line l given by parametric equations $x = 1 - 2t$, $y = 2 - t$, $z = 3t$, answer the following.

   (a) Find the equation of the plane that contains the point P and the line l.

   (b) Find the parametric equations of the line through the point P that intersects and is perpendicular to the line l.

4. (12 points) Consider the vector function

$$\mathbf{r}(t) = \langle 1 + 13\sin t, 12\cos t, 1 - 5\cos t \rangle .$$

(a) Find the speed and the normal and tangential components of acceleration at time t of the particle whose position is given by $\mathbf{r}(t)$.

(b) Compute the curvature at time t.

(c) Find the equation of the line tangent to the curve at $t = \pi$.

5. (10 points) Consider the function $f(x, y) = \sin(x)\cos(y)$.

   (a) Let $R = \{(x, y) : 0 \le x \le \pi \text{ and } 0 \le y \le \pi\}$. Find all critical points of f that lie in the region R and classify them according to type (max, min, saddle, other).

   (b) Let D be the triangular region bounded by the lines $x = \frac{\pi}{2}$, $y = \frac{\pi}{2}$, and $y = \frac{\pi}{2} - x$. Find the locations of the absolute max and min of f on D.

   (You may find the identity $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ useful.)

6. (10 points) Let $f(x, y) = x^y + y^x$.

   (a) Calculate the partial derivatives f_x and f_y.

   (b) Find the tangent plane to the surface $z = f(x, y)$ in $\mathbf{R}^3$ at the point $(1, 1, 2)$.

   (c) Use linear approximation to estimate the value of $f(1.01, 0.99)$.

7. (6 points) Consider the surface S in $\mathbf{R}^3$ given by

$$x^3 + y^3 + z^3 = 0.$$

   (a) Find the tangent plane to S at the point $(1, 0, -1)$.

   (b) Describe the shape of the intersection of S and the plane $z = 0$.

8. (10 points) Find the center of mass of the lamina that occupies the region

$$D = \{(x, y) \ : \ 4 \leq x^2 + y^2 \leq 16 \quad \text{and} \quad y \geq |x|\}$$

with density function $\rho(x, y) = y + e^{\sqrt{x^2+y^2}}$.

9. (12 points) Consider the function $f(x) = \ln(x^2 + 3x)$.

   (a) Find the Taylor series for $f(x)$ based at $b = 1$. Write your answer using sigma notation.

   (b) Find the Taylor series based at $b = 1$ for

$$F(x) = \int_1^x f(t)\, dt.$$

   Write your answer using sigma notation.

   (c) Find the 5th Taylor polynomial of $F(x)$ based at $b = 1$.

10. (12 points) Consider the function $f(x) = 2x - x^2 + e^{2x^2 - x}$.

(a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 0$.

(b) Find an upper bound on the error $|T_2(x) - f(x)|$ on the interval $[-1, 1]$.

(c) What is the smallest value of $|T_2(x) - f(x)|$ on the interval $[-1, 1]$.