Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of hand-written notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	12	
2	12	
3	12	
4	14	

Problem	Total Points	Score
5	12	
6	12	
7	12	
8	14	
Total	100	

1. (12 points) In BOTH parts, consider the three points $A(1, 1, 2)$, $B(3, 1, 5)$, and $C(1, -2, 4)$.

   (a) Find, to the nearest degree, the angle $\angle ABC$. (That is, find the angle at the corner B).

   (b) Let P be the plane that passes through A, B, and C.
   Let L be the line that passes through $(1, 0, 2)$ and $(6, 5, 7)$.
   Find the (x, y, z) point of intersection of the line L and the plane P.

2. (12 points) **NOTE: The two parts below are NOT related!**

   (a) Find all values of t at which the tangent line to the curve $x = 6 - t^3$, $y = 2t - 5t^2$ is parallel to the vector $\langle 3, 8 \rangle$.

   (b) A small bug is moving according to the vector function $\vec{\mathbf{r}}(t) = \langle t\sin(\pi t), \ln(t), t^2 - 4e^{2-2t} \rangle$. At time $t = 1$, the bug leaves the curve and follows the path of the tangent line. Find the (x, y, z) coordinates where the bug's tangent line path would intersect the xy-plane.

3. (12 points) Consider the polar curve given by the polar function

$$r = 2 + \sin(\theta).$$

Let P be the point of intersection of the polar curve with the line $y = \frac{\sqrt{3}}{3}x$ in the first quadrant.

(a) Find an equation of the tangent line to the curve at P.

(b) Find the area of the region bounded by the polar curve, the line $y = \frac{\sqrt{3}}{3}x$ and the x-axis in the first quadrant.

4. (14 points) **NOTE: The parts below are NOT related!**

   (a) Compute the partial derivatives with respect to x and y of the function $f(x, y) = x^y$.

   (b) Calculate the tangent plane to the surface defined by the equation

   $$xy\sin(z) + x - y + z = 0$$

   at the point $(x, y, z) = (2, 2, 0)$.

   (c) Find an example of a differentiable function $f(x, y)$ whose best linear approximation near $(0, 0)$ is given by the function $L(x, y) = 0$ and which has neither a local maximum nor a local minimum at $(0, 0)$.

5. (12 points) **NOTE: The parts below are NOT related!**

   (a) Find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 18$.

   (b) Evaluate the integral

$$\int_0^1 \int_x^1 \sqrt{1 + y^2}\, dy\, dx.$$

6. (12 points) The acceleration vector of a spaceship is

$$\vec{a}(t) = \langle -2\cos(t) - 3\sin(t), -\cos(t) + 6\sin(t), \sqrt{40}\cos(t)\rangle \quad \text{for all } t \geq 0$$

and the specified initial velocity and position are

$$\vec{v}(0) = \langle 3, -6, 0\rangle \qquad \vec{r}(0) = \langle 2, 1, 0\rangle.$$

(a) Find the position function $\vec{r}(t)$ of the spaceship.

(b) Find the normal component of the acceleration at time t. (Hint: The answer is a constant, simplify until you find this constant.)

7. (12 points) Consider the function $f(x) = \ln(1+3x) + xe^{-2x} - \dfrac{4x}{1+5x}$.

(a) Find the Taylor series for $f(x)$ based at $b = 0$. Write your answer using sigma Σ notation.

(b) Find the open interval on which the series in (a) converges.

(c) Find the third Taylor polynomial of $F(x)$ based at $b = 0$ where

$$F(x) = \int_0^x f(t)\,dt.$$

8. (14 points) Consider the function $f(x) = \sin(\pi x) + \cos(\pi x) + \dfrac{1}{2-x}$.

   (a) Find the second Taylor polynomial $T_2(x)$ for $f(x)$ based at $b = 1$.

   (b) Find an upper bound on the error $|T_2(x) - f(x)|$ on the interval $\left[\dfrac{1}{2}, \dfrac{3}{2}\right]$.

   (c) Find a smaller interval I centered at $b = 1$ so that the error $|T_2(x) - f(x)|$ has an upper bound 0.001 for all x in the interval I.