Your Name


Student ID \#

Professor's Name


Your Signature
$\square$


TA's Name


- CHECK that your exam contains 8 problems on 8 pages.
- This exam is closed book. You may use one $8 \frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- Place a box around YOUR FINAL ANSWER to each question.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 1 | 12 |  |
| 2 | 14 |  |
| 3 | 15 |  |
| 4 | 12 |  |


| Problem | Total Points | Score |
| :---: | :---: | :---: |
| 5 | 11 |  |
| 6 | 12 |  |
| 7 | 12 |  |
| 8 | 12 |  |
| Total | 100 |  |

1. (12 points) Consider the parametric curve $\left\langle\cos \left(t^{2}\right), \sin \left(t^{2}\right), t\right\rangle$.
(a) For which values of $t$ does the curve intersect the surface $x^{2}+y^{2}=z^{2}$ ?
(b) Are there any values of $t$ for which the binormal to the curve at time $t$ is parallel to the $z$-axis? If so, find them. If not, explain why not.
2. (14 points) Consider the polar curve given by $r=1+\sin (\theta)$.
(a) Plot this curve.
(b) Find all horizontal tangent lines to the curve.
(c) Find the area between the $x$-axis and the portion of the curve below the $x$-axis.
3. (15 points)
(a) Find an equation for the plane perpendicular to the line $x=1-t, y=t, z=14$ and containing the point $(1,1,1)$.
(b) Find an equation for the line of intersection of the planes $x+y+2017 z=2019$ and $x-y-z=2015$.
(c) Do the points $(1,2,3),(-1,0,1),(0,0,0)$, and $(\pi, 0,-\pi)$ lie on a plane? Why or why not?
4. (12 points) Consider the function $z=f(x, y)$ defined implicitly by the equation $z e^{z}=x^{2}-y^{2}$.
(a) Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
(b) Find the linearization of the function $f(x, y)$ at $(1,1)$, and use it to approximate $f(1.01,0.98)$.
5. (11 points) Find the shortest distance from the point $P(0,4,1)$ to the cone $z=\sqrt{x^{2}+y^{2}}$.
6. (12 points)
(a) Evaluate the iterated integral $\int_{0}^{1} \int_{\sqrt{x}}^{1} \sin \left(y^{3}\right) d y d x$.
(b) Find the volume of the solid (trumpet) that lies under the plane $z=20$, above the plane $z=1$, and inside the surface $z=\frac{1}{\sqrt{x^{2}+y^{2}}}$.
7. (12 points) Let $f(x)=\sin (x) e^{\left(x-\frac{\pi}{2}\right)}$.
(a) Find the third Taylor polynomial $T_{3}(x)$ based at $b=\frac{\pi}{2}$.
(b) Find an upper bound for $\left|T_{3}(x)-f(x)\right|$ on the interval $\left[\frac{\pi}{2}-0.1, \frac{\pi}{2}+0.1\right]$.
(c) Let $T_{7}(x)$ be the seventh Taylor polynomial for $f(x)$ based at $b=\frac{\pi}{2}$. What is the smallest value of $\left|T_{7}(x)-f(x)\right|$ on the interval $[0, \pi]$ ?
8. (12 points) Consider the functions $f(x)=x e^{x^{2}}-\arctan (x)$ and $F(x)=\int_{0}^{x} f(t) d t$.
(a) Find the Taylor series for $F(x)$ based at $b=0$ (use sigma notation).
(b) Find the open interval on which the series in (a) converges.
(c) Find $F^{(6)}(0)$. Give an exact answer.
