Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- CHECK that your exam contains 8 problems on 8 pages.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of notes and a TI-30X IIS calculator. Do not share notes or calculators.
- Unless otherwise specified, you should give your answers in exact form. (For example, $\frac{\pi}{4}$ and $\sqrt{2}$ are in exact form and are preferable to their decimal approximations.)
- In order to receive full credit, you must show all of your work.
- **Place a box around $\boxed{\textbf{YOUR FINAL ANSWER}}$ to each question.**
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. DO NOT USE SCRATCH PAPER.
- Raise your hand if you have a question.

| Problem | Total Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 12 | |
| 3 | 13 | |
| 4 | 13 | |
| 5 | 13 | |

| Problem | Total Points | Score |
|---|---|---|
| 6 | 12 | |
| 7 | 12 | |
| 8 | 13 | |
| Total | 100 | |

1. (12 points) Use the three points $P(1,2,1)$, $Q(3,2,2)$ and $R(-1,0,-1)$ to answer the following.

   (a) Give the parametric equations for the line that passes through the points $P$ and $Q$.

   (b) Compute $\mathbf{proj}_{\overrightarrow{PQ}}\overrightarrow{PR}$.

   (c) Compute the point on the line given in part (a) that is closest to the point $R(-1,0,-1)$.

2. (12 points) Consider the curve given by the vector function $\mathbf{r}(t) = \left\langle t^2 - 1, 2t, \frac{t^3}{3} \right\rangle$.

   (a) Compute $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

   (b) Compute a parametric equation for the tangent line to this curve when $t = 1$.

   (c) What is the curvature of $\mathbf{r}(t)$ at $t = 1$?

3. (13 points) Consider the implicitly defined surface $2xyz + xy + z^2 + 2 = xz^2 + x + y + 2z$ in $\mathbb{R}^3$.

   (a) Find the two points of intersection of the surface with the line through the origin and in direction $\mathbf{j} + \mathbf{k}$.

   (b) Write the equation of the tangent plane to the surface at the point $(0, 5, 3)$.

4. (13 points) Find the absolute maximum and minimum values of $f(x, y) = x+y+\sqrt{1-x^2-y^2}$ on the quarter disc $\{(x, y) \mid x \geq 0, y \geq 0, x^2+y^2 \leq 1\}$.

5. (13 points)

   Let $\mathcal{S}$ be the solid beneath $z = 12xy^2$ and above $z = 0$, over the rectangle $[0, 1] \times [0, 1]$.

   Find the value of $m > 1$ so that the plane $y = mx$ divides $\mathcal{S}$ into two pieces with equal volume.

6. (12 points) Compute $\displaystyle\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} (x^3 + xy^2)\, dy\, dx$.

7. (12 points) Let $f(x) = x^2 \sin(x^3) + \dfrac{1}{8 - x^3}$.

   (a) Find $T_6(x)$, the sixth Taylor polynomial for $f(x)$ based at $b = 0$.

   (b) Give the largest open interval on which the Taylor series for $f(x)$ based at $b = 0$ converges.

8. (13 points) Let $g(x) = \sqrt{3 + x^2}$.

   (a) Find $T_1(x)$, the first Taylor polynomial for $g(x)$ based at $b = 1$.

   (b) Use your answer to (a) to approximate the value of $\sqrt{3.25}$.

   (c) Use Taylor's inequality to find an upper bound for the error in your approximation in part (b).