NAME: 

SIGNATURE: 

STUDENT ID #: 

TA SECTION: 

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Instructions:

- Your exam consists of FOUR problems. Please check that you have all four of them.
- No books or notebooks allowed; you may use an A4 double-sided, handwritten sheet of notes for personal use (do not share).
- Place a box around your final answer to each question.
- No graphing calculators allowed (scientific calculators OK).
- Answers with little or no justification may receive no credit.
- Answers obtained by guess-and-check work will receive little or no credit, even if correct.
- Read problems carefully.
- Raise your hand if you have a question.
- If you need more space, use additional blank sheets which will be provided by your TA. It is your responsibility to have him/her staple the additional sheets to your exam before you turn it in.
- Please turn off cell phones. GOOD LUCK!
Problem 1. (10 pts) Find the distance of the point \( P(2, 1, 4) \) to the plane that passes through the points \( Q(1, 0, 0), R(0, 2, 0), \) and \( S(0, 0, 3) \).

Solution. The plane contains the vectors \( u := QR = \langle -1, 2, 0 \rangle \) and \( v := QS = \langle -1, 0, 3 \rangle \). Thus the distance is the absolute value of the scalar projection of \( w := QP = \langle 1, 1, 4 \rangle \) in the direction \( u \times v \). A quick calculation reveals

\[
u \times v = 6i + 3j + 2k.
\]

Thus, the required distance is given by

\[
|\text{Comp}_{u \times v} w| = \frac{17}{\sqrt{36 + 9 + 4}} = \frac{17}{7}.
\]
Problem 2. (10 pts) Find the equation of the plane that is perpendicular to the plane $x - y + z = 2$ and contains the line with symmetric equations $x = 2y = 3z$.

Solution. We need to find two vectors on the plane. One is given by the normal vector of the given plane $P(1, -1, 1)$. We get the other two from the line $Q(0, 0, 0)$ and $R(6, 3, 2)$. Let $u := PQ = <-1, 1, -1>$ and $v := PR = <5, 4, 1>$. Thus we can take the normal vector $n = u \times v = <5, -4, -9>$. Since the plane passes through $(0, 0, 0)$, the equation is

$$5x - 4y - 9z = 0.$$
Problem 3. (10 pts) Consider the parametric curve \( x = \exp (\cos(t)), \ y = \exp (\sin(t)) \), where \( 0 \leq t < 2\pi \) is the parameter.

(i) For what values of \( t \) are the tangents horizontal? For what values are the tangents vertical?

**Solution.** We have

\[
\frac{dy}{dt} = \cos(t) \exp (\sin(t)), \quad \frac{dx}{dt} = -\sin(t) \exp (\cos(t)), \quad \frac{dy}{dx} = -\frac{\cos(t)}{\sin(t)} \exp (\sin(t) - \cos(t)).
\]

Tangent horizontal at \( t = \{\pi/2, 3\pi/2\} \) and vertical at \( t = \{0, \pi\} \).

(ii) Write down the equation of the tangent line to this curve at \( t = \pi/4 \).

**Solution.** Slope is \(-1\). Thus the line is

\[ y = -x + 2 \exp(1/\sqrt{2}). \]
Problem 4. (10 pts) Consider the polar curve

\[ r + \frac{c}{r} = \sin(\theta) + \cos(\theta) \]

where \( c \) is some constant.

(i) Show that for \( c > 1/2 \) this equation has no solution.

Solution. We write it as \( r^2 + c = r\cos \theta + r\sin \theta \). That gives us

\[ x^2 + y^2 + c = x + y. \]

Completing the square we get

\[ (x - 1/2)^2 + (y - 1/2)^2 = 1/2 - c. \]

No solution when \( c > 1/2 \) since the left side is positive (or zero) while the right side is negative.

(ii) Show that for \( c \leq 1/2 \) the above curve is a circle. What is the center and the radius of this circle?

Solution. Center at \((1/2, 1/2)\) and radius \(\sqrt{1/2 - c}\).
Extra sheet.