MIDTERM 1

MATH 126 **Last name, first name:** ______________________

Section: __________

Student number: ______________

Signature: ______________

Please do not start working until instructed to do so.

You have 50 minutes.

Please show your work.

Scientific, but not graphing calculators are OK.

You may use one 8.5 by 11 sheet of handwritten notes.

Problem 1. *(8 points total)* Determine whether the statements are true or false. (Clearly say: TRUE or FALSE.) There is no partial credit for this problem.

a. *(2 points)* The Taylor series for $f(x) = \cos(4x^2)$ centered at $x = 0$ is convergent for all x.

b. *(2 points)* The length of the sum of two vectors is always equal to the sum of the lengths of the vectors.

c. *(2 points)* If the Taylor series for $f(x)$ centered at $x = -3$ converges at $x = 10$ then it converges at $x = -8$.

d. *(2 points)* $\mathbf{u}\cdot(\mathbf{v} \times \mathbf{w}) = (\mathbf{u}\cdot\mathbf{v}) \times (\mathbf{u}\cdot\mathbf{w})$ for any vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ in $\mathbb{R}^3$.

Problem 2. *(11 points total)* Consider the function $f(x) = \sin\left(\frac{\pi x}{6}\right)$.

a. *(6 points)* Find $T_2(x)$, the second order Taylor polynomial for $f(x)$ centered at $a = 1$.

b. *(5 points)* Use Taylor's inequality fo find an upper bound on $|f(1.1) - T_2(1.1)|$.

Problem 3. *(12 points total)* Let L_1 be the line given by the parametric equations

$$x = 2t, y = 0, z = 4 - 4t,$$

and let L_2 be the line given by the parametric equations

$$x = 2 - 2u, y = 3u, z = 0.$$

a. *(6 points)* Find the point of intersection of L_1 and L_2.

b. *(6 points)* Find an equation of the plane that contains both L_1 and L_2. Give your answer in the form $ax + by + cz = d$.

Problem 4. *(11 points)* Consider the function $f(x) = \dfrac{x}{(3+x^2)^2}$. Using the fact that

$$\left(\frac{1}{3+x^2}\right)' = -2f(x)$$

find the Taylor series for $f(x)$ centered at $x = 0$ and find its radius of convergence.

Problem 5. *(8 points)* Find a vector $\mathbf{u}$ which satisfies both of the following conditions:

(i) $\mathbf{u}$ is orthogonal to $\langle 2, 1, 4 \rangle$,

(ii) the cross product of $\mathbf{u}$ and $\langle 1, 2, 0 \rangle$ equals $\langle 2, -1, 0 \rangle$.