Math 126A/B **Exam #1** **19 April 2007**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and put away all electronic devices except your non-graphing calculator.
- This exam is closed book. You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes (**only one side** may be used). Do not share notes.
- Give your answers in exact form. Do not give decimal approximations unless they are specifically requested.
- In order to receive credit, you must show your work on the exam paper, with some explanation in English, if appropriate. Do not do computations in your head. Instead, write them out on the exam paper.
- **Place a box around YOUR FINAL ANSWER to each question.**
- If you need more room, use the **back of the previous page** and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. Let $\mathbf{a} = \langle 1, 1, x \rangle$ and $\mathbf{b} = \langle 0, x, 1 \rangle$.

(a) [**5 points**] Find $x \in \mathbb{R}$ such that $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.

(b) [**5 points**] Find $x \in \mathbb{R}$ such that the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\dfrac{\pi}{4}$.

2. Let A be the point $(1, 2, 3)$ and O the origin $(0, 0, 0)$. Consider the points $P(x, y, z)$ such that

$$\overrightarrow{AP} \cdot \overrightarrow{OP} - \overrightarrow{OA} \cdot \overrightarrow{OP} = 2 - \left|\overrightarrow{OA}\right|^2 .$$

Show that the set of all such points is a sphere, and find its center and radius.

3. Let $f(x) = \cos\left(\frac{\pi}{3}x\right)$.

(a) [**5 points**] Find $T_2(x)$, the second Taylor polynomial for $f(x)$, centered at $b = \frac{1}{2}$.

(b) [**5 points**] Find the error bound for the approximation given by T_2 on the interval $I = [\,0, 1\,]$. You may give your answer in decimal form if all steps are fully justified.

4. Find the Taylor series for the function

$$f(x) = 4\int_0^x \sin t \cos t \, dt + x^2 \ln(1 - x^2)$$

based at $b = 0$.

5. Let $f(x) = e^{x+1} + \dfrac{1}{2x+3}$.

(a) [**6 points**] Find the Taylor series for $f(x)$ based at $b = 1$.

(b) [**4 points**] Give an interval on which the Taylor series you found in part (a) converges. You may specify the interval in decimal form if all steps are fully justified.