MATH 126 Midterm Exam 2
Tuesday November 23, in your quiz section

Name: | | Quiz section: | |

7-digit UW ID: | |

Exam Instruction:

- You have 50 minutes to complete the 5–page exam. Distribute your time wisely.
- Show your work to earn full credit.
- **Do NOT write within 1 cm of the edge!** Your exam will be scanned for grading.
- Leave your answer in the exact form rather than a decimal approximation. For example, you may leave your answer as a fraction, a square root expression, an inverse trig function of an expression, etc.
- You can prepare one hand-written double-sided 8.5” $\times$ 11”–inch page of notes and bring it to the exam.
- You may use a basic calculator that can not graph, differentiate, or integrate (e.g., TI-30X IIS). **All other electronic devices (e.g., cell phone, earbuds) are forbidden.**
- You must finish the exam independently. **Giving or receiving any assistance on the exam is considered cheating, which will result in a grade of zero for the exam.** There are multiple versions of the exam, do not discuss the exam questions with other students on the exam day.
- **Please tear off the last page of scratch paper. Do NOT turn in the scratch paper unless you have put work on it to be graded.**

1. (12 pts) Let $z = f(x, y)$ be a function defined by the implicit equation

$$xz + e^z = y^2$$

(a) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

(b) Use your answer in (a) to give a vector tangent to the trace of the surface $z = f(x, y)$ on the plane $x = 2$ at the point $(2, 1, 0)$. (The question did not ask for the tangent line, just give a tangent vector.)

2. (15 pts) Consider the function $f(x, y) = x^2 + \frac{1}{3}y^3 - 3y$.

(a) Find all the critical points of f in $\mathbb{R}^2$, then determine if f has a local maximum, local minimum, or a saddle point at each of the critical point. (No need to find the f value at the critical points for this part.)

(b) Find the absolute maximum and absolute minimum value of f on the disk $D = \{(x, y) \,|\, x^2 + y^2 \leq 1\}$.

3. (12 pts) Consider the integral

$$\int_0^1 \int_{2x}^2 \cos(y^2)\, dy dx$$

(a) Sketch and shade the region of integration on the xy–plane.

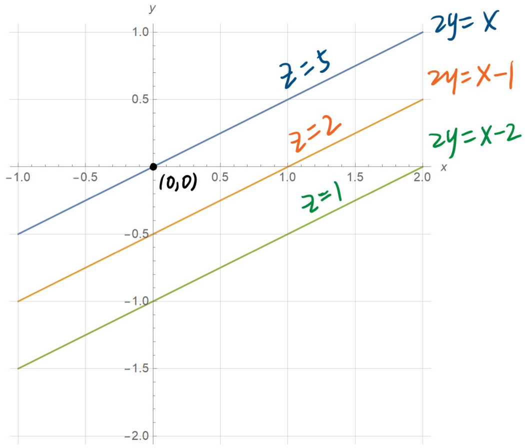
(b) Reverse the order of integration and evaluate the integral.

4. (10 pts) Setup a double integral in **Polar coordinate** to find the volume of the solid below the cone $z = 6 - \sqrt{x^2 + y^2}$ and above the paraboloid $z = x^2 + y^2$. Do **NOT** evaluate the integral.

$$\int_{\square}^{\square} \int_{\square}^{\square} \boxed{\qquad\qquad\qquad} \, d\square \, d\square$$

To get full and/or partial credit, show work to arrive the upper/lower limits of the integral.

5. (11 pts) The graph below illustrated 3 level curves of an unknown function $z = f(x, y)$. The lines are: $2y = x$ with $z = 5$; $2y = x - 1$ with $z = 2$; $2y = x - 2$ with $z = 1$.



(a) Give a reasonable (and as precise as possible) estimation for $f_x(0, 0)$ and $f_y(0, 0)$.

(b) Based on the given level curves, would you say the second derivative $f_{xx}(0, 0)$ is positive or negative? Briefly explain your answer.

(c) (**This is a challenge question that worth 3 points, do not spend much time on this question unless you have confidently done the other problems.**)

Find a function $z = f(x, y)$ that has the given three level curves. The answer is not unique.

Please tear off this page of scratch paper. Do NOT turn in the scratch paper unless you have put work on it to be graded.